Double Rotating Angled Flanges for Adjustable Diagnostic Pointing

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Introduction

It is relatively common for diagnostics and optics on high-power laser experiments to need to be mounted onto the outside of the target chamber at an angle. This is generally achieved by manufacturing a custom angled flange to hold the device at the correct angle. This is extremely wasteful, however, as these flanges are usually large, difficult to store, and only usable by a particular diagnostic at a specific angle (preventing them from being reused). It is therefore desirable to design a reusable angle flange, compatible with typical port dimensions, that is capable of continuous angular adjustment.

Design Principle

The solution method chosen is to construct a two-part, "double rotating" flange, with each part featuring two circular faces at an angle α to one another. We shall refer to these parts as 'wedges'. As an indicative model, each wedge can be modelled as a sector of a torus, with solid radius r and radius of revolution R (see Figure 1). By stacking two of these wedges on top of one another, we can intuitively form a combined flange with a combined angle of 2α, or a combined angle of 0 if the two flanges are reversed (see Figure 2). Since these two wedges share a common circular face, rotating one flange relative to the other enables us to obtain a 'combined angle' anywhere between these two extremes. Thus, this principle allows for continuous angular adjustment of the flange.

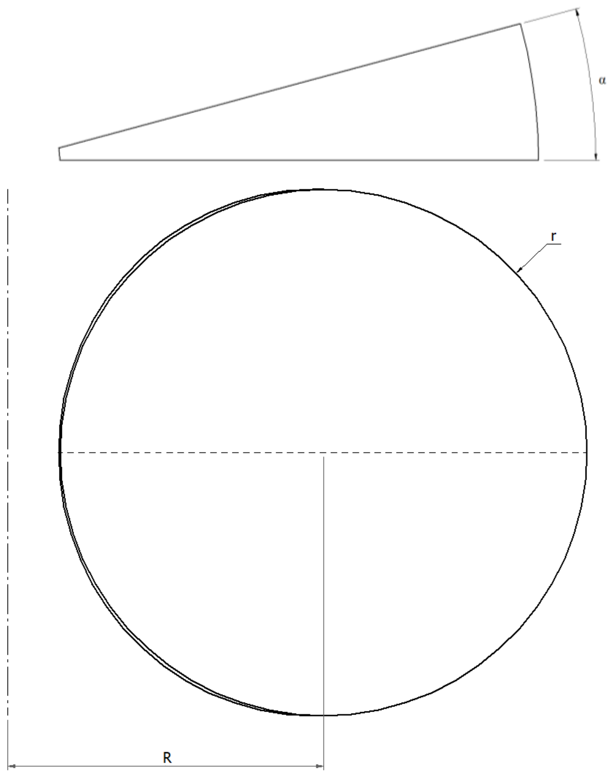
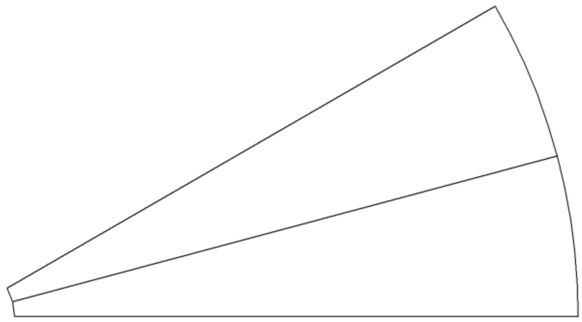
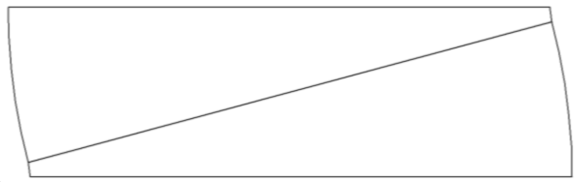


Figure 1: *Indicative model of a 'wedge' as a torus sector*



a)



b)

Figure 2: *Two wedges stacked so as to produce combined angles of 2α (a) and 0 (b).*

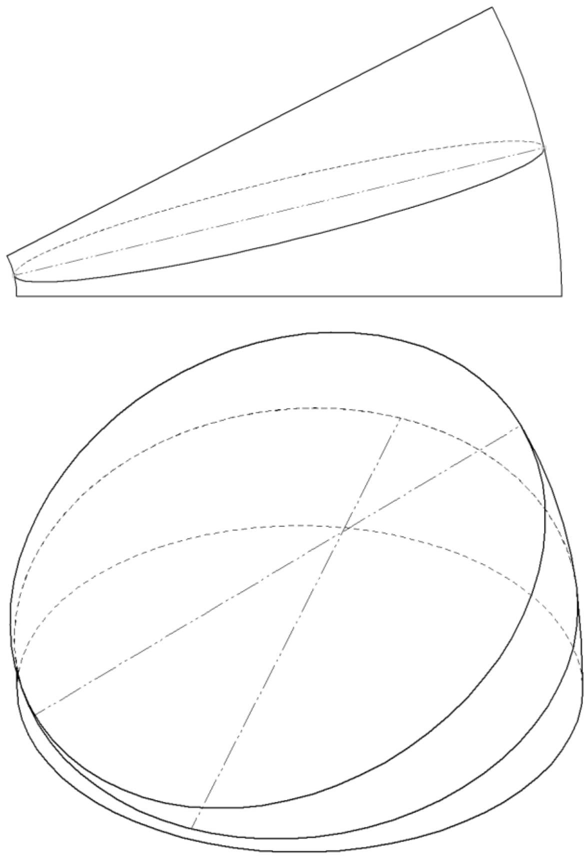


Figure 3: *Obtaining an arbitrary overall angle, between the two extremes, by rotating the two wedges relative to one another.*

Angular Mathematics

It is useful to be able to mathematically deduce the overall angle that will be formed, γ, as a result of rotating one wedge by a specified angle, β, relative to the other. This can help inform how to set up the double-angle flange to achieve a particular diagnostic angle. To do this, we first need to mathematically define the planes of the two faces of each wedge.

Planes are defined in terms of a point, $\boldsymbol{P}$, on a plane, and a unit vector $\boldsymbol{n}$ perpendicular to that plane. If we define one face of our wedge as lying on the xz plane centred on the origin, its plane definition is trivial:

$$\boldsymbol{P_0} = \begin{bmatrix} \mathbf{0} \\ \mathbf{0} \\ \mathbf{0} \end{bmatrix}$$

$$\boldsymbol{n_0} = \begin{bmatrix} \mathbf{0} \\ \mathbf{1} \\ \mathbf{0} \end{bmatrix}$$

We can define the other face of the wedge fairly simply by considering the point $\boldsymbol{P_1}$ on this plane to be directly above the origin, as shown in Figure 4.

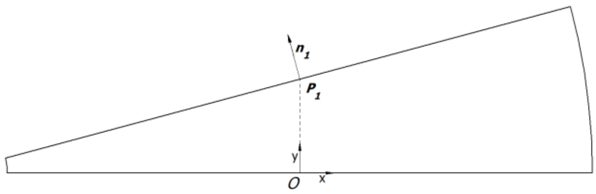


Figure 4: *Defining the top and bottom planes of a wedge*

By simple trigonometry, we can show that:

$$\boldsymbol{P_1} = \begin{bmatrix} 0 \\ R\tan\alpha \\ 0 \end{bmatrix}$$

$$\boldsymbol{n_1} = \begin{bmatrix} -\sin\alpha \\ \cos\alpha \\ 0 \end{bmatrix}$$

Let us now imagine adding our second wedge, such that the two share the xz plane as a common face, but with the second wedge offset from the first by some angle β, as per Figure 5.

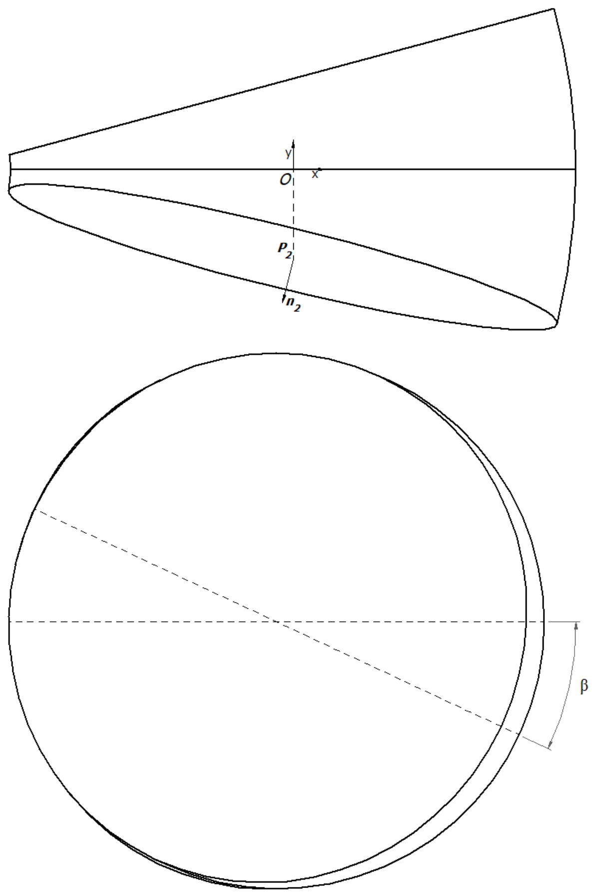


Figure 5: *The addition of a second wedge, sharing the xz plane*

We can think of the second wedge as having been flipped in the xz plane relative to the first one, and then rotated around the origin. It thus follows that its other face can be defined in terms of a point, $\boldsymbol{P_2}$, that is simply a reflection of $\boldsymbol{P_1}$ in the xz plane, and a vector $\boldsymbol{n_2}$ that is a reflection of $\boldsymbol{n_1}$ multiplied by a rotation vector of angle β about the y axis. In mathematical terms:

$$\boldsymbol{P_2} = \begin{bmatrix} 0 \\ -R\tan\alpha \\ 0 \end{bmatrix}$$

$$\boldsymbol{n_2} = \boldsymbol{R_{yB}} \times \begin{bmatrix} -\sin\alpha \\ -\cos\alpha \\ 0 \end{bmatrix}$$

$$\boldsymbol{n_2} = \begin{bmatrix} \cos\beta & 0 & \sin\beta \\ 0 & 1 & 0 \\ -\sin\beta & 0 & \cos\beta \end{bmatrix} \begin{bmatrix} -\sin\alpha \\ -\cos\alpha \\ 0 \end{bmatrix} = \begin{bmatrix} -\sin\alpha\cos\beta \\ -\cos\alpha \\ \sin\alpha\sin\beta \end{bmatrix}$$

In order to find the overall angle between the top and bottom planes of the double-wedge, we first need to define the plane in which this overall angle will lie (in other words, the angle at which we cannot see any curvature on the top and bottom faces of the double wedge). We shall refer to this as the 'viewing plane', defined by the normal vector $\boldsymbol{n_3}$. This viewing plane will be the plane mutually perpendicular to the top and bottom planes. This plane's normal vector can be found by taking the cross product of the normal vectors for the top and bottom planes:

$$\boldsymbol{n_3} = \boldsymbol{n_1} \times \boldsymbol{n_2} = \begin{bmatrix} -\sin\alpha \\ \cos\alpha \\ 0 \end{bmatrix} \times \begin{bmatrix} -\sin\alpha\cos\beta \\ -\cos\alpha \\ \sin\alpha\sin\beta \end{bmatrix}$$

$$\boldsymbol{n_3} = \begin{bmatrix} \cos\alpha\sin\alpha\sin\beta \\ \sin^2\alpha\sin\beta \\ \cos\alpha\sin\alpha\,(1+\cos\beta) \end{bmatrix}$$

Simplifying:

$$\boldsymbol{n_3} = \begin{bmatrix} \sin\beta \\ \tan\alpha\sin\beta \\ 1+\cos\beta \end{bmatrix}$$

The next step is to define a pair of vectors lying in this plane which we can measure an angle between. To keep the mathematics relatively simple, it is easiest to have one of these vectors ($\boldsymbol{v_1}$) lying along the top face, and one ($\boldsymbol{v_2}$) lying along the xz plane, where the two wedges meet. The overall angle, γ, of the combined flange will be equal to twice the angle between these two vectors, as shown in Figure 6.

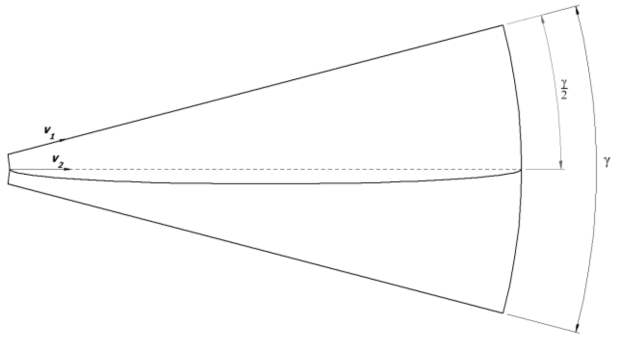


Figure 6: *The combined double-wedge from the direction of the viewing plane, with vectors and angles annotated.*

We can find the formula for $\boldsymbol{v_1}$ by taking the cross product of the normal vectors of the two planes that are intersecting, the viewing plane and the top face:

$$\boldsymbol{v_1} = \boldsymbol{n_1} \times \boldsymbol{n_3}$$

$$\boldsymbol{v_1} = \begin{bmatrix} -\sin\alpha \\ \cos\alpha \\ 0 \end{bmatrix} \times \begin{bmatrix} \sin\beta \\ \tan\alpha\sin\beta \\ \cos\beta+1 \end{bmatrix}$$

$$\boldsymbol{v_1} = \begin{bmatrix} \cos\alpha\,(\cos\beta+1) \\ \sin\alpha\,(\cos\beta+1) \\ -\tan\alpha\sin\beta\sin\alpha - \cos\alpha\sin\beta \end{bmatrix}$$

$$\boldsymbol{v_1} = \begin{bmatrix} \cos\beta+1 \\ \tan\alpha\,(\cos\beta+1) \\ -\sin\beta\,(1+\tan^2\alpha) \end{bmatrix}$$

We can work out $\boldsymbol{v_2}$ similarly, with the intersecting planes in this case being the viewing plane and the xz plane:

$$\boldsymbol{v_2} = \begin{bmatrix} \mathbf{0} \\ \mathbf{1} \\ \mathbf{0} \end{bmatrix} \times \boldsymbol{n_3}$$

$$\boldsymbol{v_2} = \begin{bmatrix} \mathbf{0} \\ \mathbf{1} \\ \mathbf{0} \end{bmatrix} \times \begin{bmatrix} \sin\beta \\ \tan\alpha\sin\beta \\ \cos\beta+1 \end{bmatrix} = \begin{bmatrix} \cos\beta+1 \\ 0 \\ -\sin\beta \end{bmatrix}$$

Finally, we can measure the angle between these two vectors using the following formula:

$$\cos\frac{\gamma}{2} = \frac{\boldsymbol{v_1}\cdot\boldsymbol{v_2}}{|\boldsymbol{v_1}||\boldsymbol{v_2}|}$$

$$\cos\frac{\gamma}{2} = \frac{(\cos\beta+1)^2 + 0 + \sin^2\beta\,(1+\tan^2\alpha)}{\sqrt{[(1+\tan^2\alpha)(\cos\beta+1)^2 + \sin^2\beta\,(1+\tan^2\alpha)^2][(\cos\beta+1)^2+\sin^2\beta]}}$$

This ultimately rearranges to:

$$\cos\frac{\gamma}{2} = \sqrt{\frac{\frac{(\cos\beta+1)^2}{\sin^2\beta} + (1+\tan^2\alpha)}{(1+\tan^2\alpha)\left[\frac{(\cos\beta+1)^2}{\sin^2\beta}+1\right]}}$$

Or:

$$\gamma = 2\cos^{-1}\sqrt{\frac{(p+q)}{p(q+1)}}$$

Where:

$$p = (1+\tan^2\alpha)$$

$$q = \frac{(\cos\beta+1)^2}{\sin^2\beta} = \cot\frac{\beta}{2}$$

Alternatively we can express this in terms of the rotation angle β required to give an overall angle γ:

$$\beta = 2\cot^{-1}\sqrt{\frac{p - p\cos^2\frac{\gamma}{2}}{(p\cos^2\frac{\gamma}{2} - 1)}}$$

The crucial thing to note is that this equation (and its derivation) is independent of either r or R, or indeed any function of the actual wedge geometry. This means that, so long as the angle of each wedge remains the same, they can be any shape that we desire. Note also that although the combined flange angle is independent of the orientation of the flange about its mounting point, this flange mounting angle will determine the pointing of the flange, and so the flange must be able to rotate in this axis too for it to be usable.

The Design

The first flange designed according to this double rotating principle was a low-profile design, capable of mounting an ISO100 tube fitting and diagnostics up to 100mm in diameter at angles of up to 30 degrees from the normal. It mounts directly to a 250mm diameter Gemini port. The two wedges are held together, and to the door, using a clamp ring, allowing the flanges to rotate relative to one another and relative to the port they mounted onto when the ring is loosened. The equation relating rotating angle to overall flange angle is used to define a scale etched around one of the wedges, showing how the two flanges should be oriented to achieve a particular overall angle. This flange was first used to mount an angled kapton window for the Gemini TA3 Higginbotham experiment in February 2020.

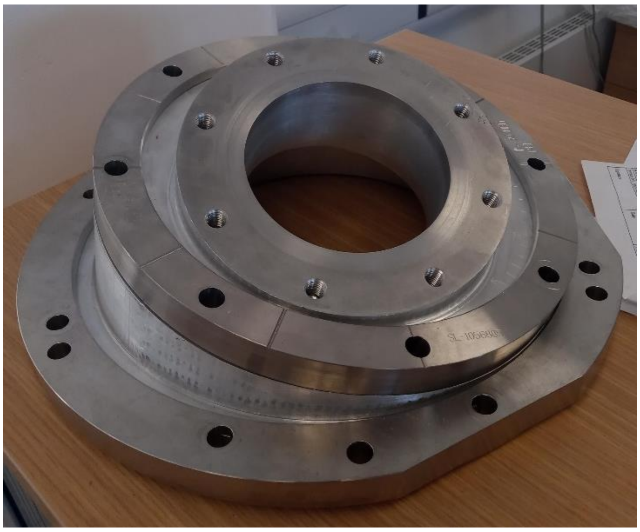
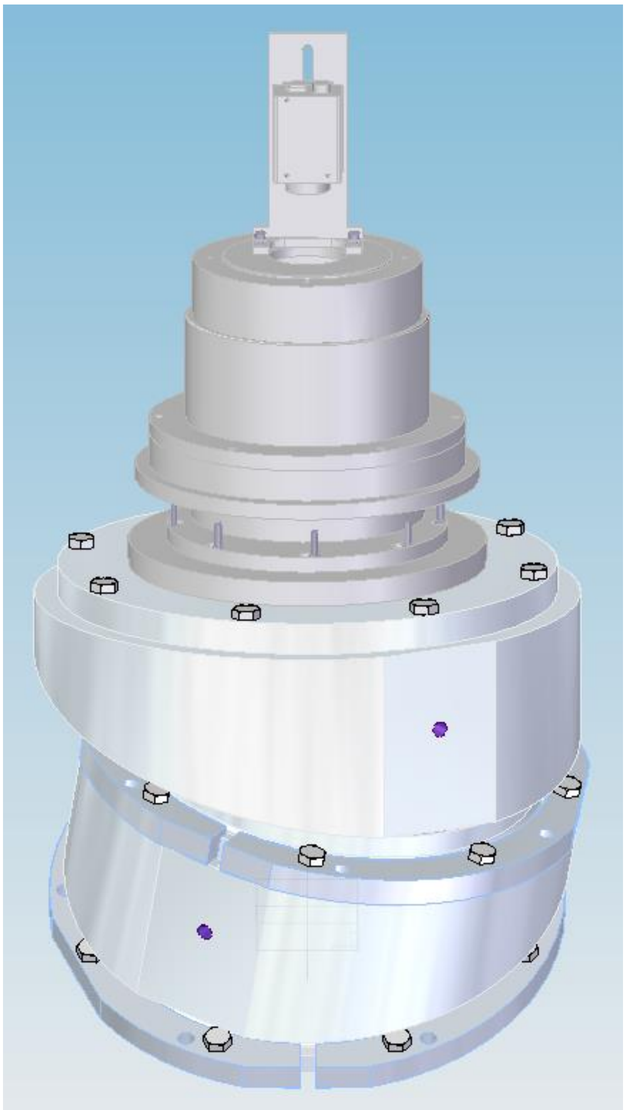


Figure 7: *The low-profile double-rotating flange.*

This flange was, however, too small to accommodate the streak camera required for the Gemini TA3 Kettle experiment. It is also difficult to machine to a good quality. Therefore, a second double-angle flange has also been designed, referred to as the ‘armadillo’ flange. The wedges of this flange are larger, but are each capable of being machined (with the help of a jig) in just two turning operations. They are also held together using clamp rings. It can also accommodate diagnostics up to 150mm diameter and at up to 35 degrees from the normal, at the expense of being significantly bulkier & heavier than the low-profile design. For this reason, it also incorporates crane lifting points into each wedge.



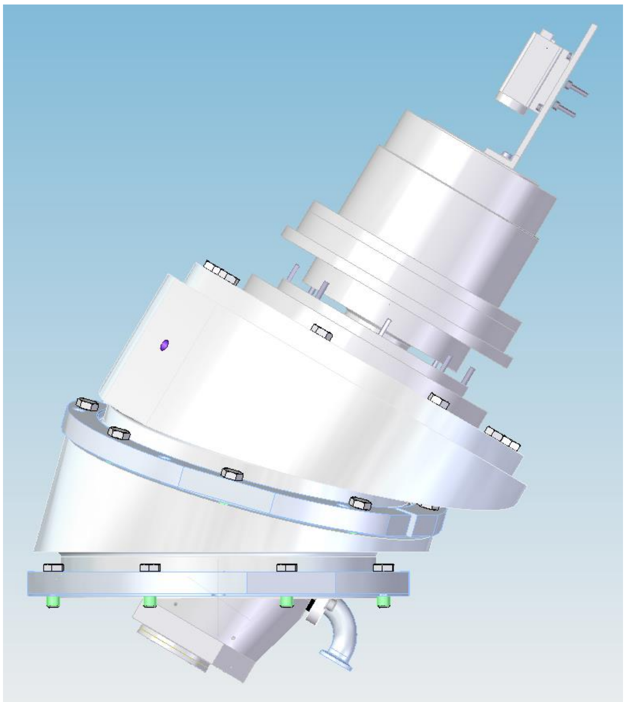


Figure 8: *CAD model of the 'armadillo' profile double-rotating flange, with streak camera mounted into it.*

Conclusions

Though these flanges have yet to see widespread service, they should enable the accurate and adjustable positioning of a wide range of diagnostics used on the CLF high-power laser facilities while minimizing the need to use costly and wasteful custom angled flanges.

Acknowledgements

The author would like to credit Daniel Treverrow, who first came up with the double-rotating flange concept and did the initial design work to realise the low-profile design. He would also like to thank Mark Dearing for his comments on the low-profile flange design, and his advice on both machining the 'armadillo' flange and design of a suitable turning jig.