The effect of quantum radiation reaction on electron motion in ultra-high intensity laser-matter interactions

Contact: christopher.ridgers@york.ac.uk

C. P. Ridgers
York Plasma Institute,
Physics Department, University of York,
YO10 5DD, United Kingdom

1 Introduction

Experiments at todays intensity frontier (2×10^{22}Wcm^{-2} achieved by the Hercules laser at the University of Michigan [1]) are on the verge of generating a QED-plasma in the laboratory for the first time. The QED-plasma regime is reached when the lasers electric field in the electrons rest frame E_{RF} approaches the Schwinger field E_s required to break down the vacuum into electron-positron pairs, $\eta = E_{RF}/E_s \sim 0.1(I/5 \times 10^{22}\text{Wcm}^{-2}) > 0.1$ [2, 3, 4]; I is the laser intensity. The defining feature of the QED-plasma regime is that non-linear QED emission processes strongly affect the plasma dynamics. Two processes, highlighted below, are dominant. The electrons in the plasma are accelerated so violently by the electromagnetic fields of the laser pulse that they radiate a significant fraction of their energy as gamma-ray (MeV energy) photons by strongly non-linear inverse Compton scattering [3, 4, 5]. Therefore, the radiation reaction force [6], usually neglected, must be included in their equation of motion: the radiation-damped QED plasma regime is reached. Furthermore, at the intensity where radiation reaction comes into play quantum effects also start to become important. Radiation-reaction is dramatically reduced relative to the classical theory and becomes probabilistic (because of this radiation reaction will be referred to in what follows as a QED effect). The latter necessitates a radically new plasma model that replaces the current foundational idea of an electron moving along a deterministic classical worldline with a probability density evolving in phase-space.

As the intensity becomes higher, $\eta \sim 1$ and the emitted gamma-ray photons have a high probability of generating electron-positron pairs in the laser-fields by the multi-photon Breit-Wheeler process. These pairs can go on to emit further photons which generate further pairs and a cascade of pair production ensues which can result in extremely dense (up to solid-density) pair plasmas: the QED-plasma becomes pair-dominated. The upgrade to the Texas Petawatt Laser (completion in 2016) is projected to reach 5×10^{22}Wcm^{-2} and should reach the radiation-damped QED-plasma regime. ELI-beamlines should also reach this level and ELI-Nuclear Physics (ELI-NP) should reach intensities $\sim 10^{24}$Wcm^{-2} and so the pair-dominated regime by $\sim$2018.

2 Quasi-Classical Kinetic Equation Describing Electron Motion

I will now discuss how to describe electron motion in a QED-plasma were electrons are subject to a stochastic radiation reaction force. I use the quasi-classical model of Baier & Katkov whereby the motion of the electron is treated classically between emission events [7]. The rates of the QED processes are calculated in the strong-field QED framework [8] where the laser (& plasma) electromagnetic fields are treated as a classical background field. I make the following assumptions about this background field. The ratio of the photon formation length to the laser wavelength is equal to $1/a_0$. I will consider interactions where $a_0 \gg 1$ and so the laser fields can be assumed constant on the length-scale over which emission occurs: the background fields are quasi-static. In addition, the laser fields are much less than E_s and so the background field is weak. These assumptions allow us to assume that: (i) the emission rate depends only on the local fields; (ii) the characteristics of the field are unimportant in determining the emission rate.

In this case the rate of photon emission by an electron of energy $\gamma m_e c^2$ is given by

$$\lambda_\gamma = \frac{\sqrt{3}\alpha_f c}{\lambda_c}\frac{\eta}{\gamma}h(\eta) \tag{1}$$

λ_c is the Compton wavelength. $h(\eta) = \int_0^{\eta/2} d\chi F(\eta,\chi)/\chi$ where $F(\eta,\chi)$ is the quantum-corrected synchrotron spectrum as given by Erber [9] and Sokolov & Ternov [10]. $\chi = (e\hbar^2 2m_e^3c^4)|F^{\mu\nu}k_\nu|$ parameterises the photon energy (k_ν is its 4-wavevector). The modification of $F(\eta,\chi)$ away from the classical synchrotron spectrum leads to a quantum correction to the instantaneous power radiated $\mathcal{P} = (4\pi m_e c^3/3\lambda_c)\alpha_f\eta^2 g(\eta) = \mathcal{P}_c g(\eta)$.

If we consider the case, relevant to laser-matter interactions, of an electron counter-propagating relative to a circularly-polarised plane electromagnetic wave we may define the probability that it the electron has Lorentz factor γ at time t as $\Phi(\gamma,t)$. This probability obeys the following equation [11, 12].

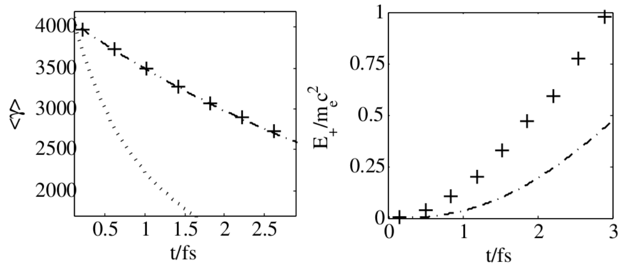

Figure 1: Left: time evolution of the average energy for an ensemble of 10^5 electrons with initial energy $4000 m_e c^2$ counter-propagating relative to a circularly-polarised plane wave with $a_0 = 20$ (crosses). Comparison is made to the deterministic solution of equation (4) (dot-dashed line) and the solution of the classical equation, identical to equation (4) except for the omission of $g(\eta)$ (dotted line). Right: identical plot describing positron production.

$$\frac{\partial \Phi_{\pm}(\gamma,t)}{\partial t} = -\lambda_{\gamma}(\eta)\Phi_{\pm}(\gamma,t) + \int_{\gamma}^{\infty} d\gamma' \lambda_{\gamma}(\eta') p_{\chi}(\eta',\chi)\Phi_{\pm}(\gamma',t) \quad (2)$$

$p_{\chi}(\eta,\chi) = [1/h(\eta)][F(\eta,\chi)/\chi]$ is the probability that an electron with parameter η emits a photon with χ.

3 Comparision to the Classical Equation of Motion

Multiplying equation (2) by γ and integrating over γ yields [12]

$$\frac{d\langle\gamma\rangle}{dt} = -\left\langle \frac{\mathcal{P}(\eta)}{m_e c^2} \right\rangle \quad (3)$$

For comparison the equation of motion for a particle radiating in a deterministic fashion is

$$\frac{d\gamma_d(t)}{dt} = -\frac{\mathcal{P}[\eta_d(t)]}{m_e c^2} \quad (4)$$

$\gamma_d(t)$ & $\eta_d(t)$ are the Lorentz factor and η-parameter of the electron moving on a deterministic worldline. To arrive at this equation we have followed the Landau & Lifshitz prescription for dealing with the radiation reaction force [6] and taken the ultra-relativistic limit. We have also made the substitution $\mathcal{P}_c \rightarrow \mathcal{P}$ [2], thus capturing the quantum reduction in the synchrotron power but not the stochasticity of the emission. Henceforth this will be described as the 'deterministic' emission model, as opposed to the 'probabilistic' model which includes the quantum stochasticity.

In the classical limit the variance in Φ_- is small, $\Phi_- \rightarrow \delta[\gamma - \gamma_d(t)]$, $d\langle\gamma\rangle/dt \rightarrow d\gamma_d/dt$ and $\langle\mathcal{P}(\eta)\rangle \rightarrow \mathcal{P}[\eta_d(t)]$, demonstrating correspondence between the probabilistic equation (3) and the deterministic equation (4). Figure 1 shows that the deterministic theory predicts the average energy loss of the electrons even in the case where $\eta \sim 1$. This surprising result can be explained by the fact that for $\eta \sim 1$, $\mathcal{P}$ is approximately proportional to η. Figure 1 also shows that although the determinisitc model accurately predicts the rate of electron energy loss and so gamma-ray photon production, it is not accurate for predicting the rate of positron production.

4 Conclusion

To describe the dynamics of QED-plasmas created by next generation lasers radiation reaction needs to be included in the electron equation of motion. Here I have shown that this can be done by a relatively straightforward modification of the equation of motion: the inclusion of a determinisitic form of the radiation reaction force (reduced by g relative to the classical force). This will be extremely useful when one attempts to make quantitative theoretical predictions about the behaviour of QED-plasmas, for example in deriving a simple theory of absorption, as we may simply augment existing derivations for classical relativistic plasmas which use a deterministic equation of motion. As intensities increase further and we reach a regime where pair production is important we cannot use a determinisitic model and a more radical re-evaluation of existing theory will be required.

References

[1] V. Yanovsky *et al*, Opt. Express, **16**, 2109 (2008)

[2] J.G. Kirk, A.R. Bell & I. Arka, Plas. Phys. Control Fusion, **51**, 085008 (2009)

[3] E.N. Nerush *et al*, Phys. Rev. Lett., **106**, 035001 (2011)

[4] C.P. Ridgers *et al*, Phys. Rev. Lett., **108**, 165006 (2012); C.S. Brady *et al*, Phys. Rev. Lett., **109**, 245006 (2012); C.P. Ridgers *et al*, Phys. Plasmas, **20**, 056701 (2013)

[5] A. Zhidkov *et al*, Phys. Rev. Lett., **88**, 185002 (2002); T. Nakamura *et al*, Phys. Rev. Lett., **108**, 195001 (2012)

[6] L.D. Landau & E.M. Lifshitz, The Course of Theoretical Physics Vol 2. (Butterworth-Heinemann, Oxford, 1987), p222-229

[7] V.N. Baier & V.M. Katkov, Phys. Lett., 25A, 492 (1967); Zh. Eksp. Teor. Fiz., 53, 1478 (1967)

[8] W.H. Furry, Phys. Rev., **81**, 115 (1951)

[9] T. Erber, Rev. Mod. Phys., **38**, 626 (1966)

[10] A.A. Sokolov & I.M. Ternov, 'Synchorotron Radiation', Akademie-Verlag, Berlin, 1968

[11] C.S.Shen & D. White, Phys. Rev. Lett., **28**, 455 (1972)

[12] C.P. Ridgers *et al*, J. Comput. Phys., **260**, 273 (2014)