Aspects of electron acoustic wave physics in laser backscatter

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Introduction

Recent single hot-spot experiments on the Trident laser system[1,2] identified SRS-like backscatter from an electron plasma mode whose frequency is significantly below the plasma frequency. This mode was identified as the Electron Acoustic Wave (EAW), an undamped mode supported by the trapping of electrons. Here we outline the theoretical background to the EAW, and simulate the mode to demonstrate its undamped nature by utilising an Eulerian Vlasov code. We discuss some of the implications for Laser Plasma Interactions (LPI) that may affect reflectivity, including Stimulated Electron Acoustic Scattering (SEAS) and the Langmuir Decay Instability (LDI).

The Electron Acoustic Wave

The existence of plasma waves at frequencies below the electron plasma frequency was first identified by Stix[3], although it was expected that Landau damping in this regime would prohibit their formation. Later work[4] showed that EAWs can exist, undamped, supported by a population of trapped electrons.

The conventional derivation of the plasma dispersion relations, based on a two-fluid treatment, yields high frequency Langmuir waves and low frequency ion-acoustic waves. A more complete kinetic treatment, based on linearising the Vlasov equation, shows these modes to be damped. This is the phenomenon of Landau damping, a purely kinetic effect which requires that $\partial_v f|_{v_p} < 0$, where *f* is the particle distribution function and v_p the phase velocity of the wave. However, if $\partial_v f|_{v_p} = 0$ then the wave may be undamped. This is the case for the EAW.

In order to construct a dispersion relation for the EAW we first consider a Maxwellian distribution with a flattened region at $v=v_p$ in the limit where the width of the flattened region, in velocity space, tends to zero while $\partial_v f|_{v_p} = 0$. For a real frequency ω the integral in the Landau dispersion relation

$$\varepsilon(\omega,k) = 1 - \frac{1}{2(k\lambda_d)^2}\int_{-\infty}^{+\infty}\frac{\partial_v f(v)}{v-\omega/(kv_T)}dv \quad (1)$$

can be written

$$P\int_{-\infty}^{+\infty}\frac{\partial_v f(v)}{v-\omega/(kv_T)}dv + i\pi\partial_v f|_{v_p}. \quad (2)$$

By construction, the second term is zero, leaving only the principal value integral. Evaluating this integral gives a dispersion relation for EAWs in the linear limit.

$$k^2\lambda_d^2 + 1 - \sqrt{2}\frac{\omega}{k}Daw\left(\frac{\omega}{\sqrt{2}k}\right) = 0 \quad (3)$$

$$Daw(t) = e^{-t^2}\int_0^t e^{u^2}du \quad (4)$$

where Equation (4) is the Dawson integral. Evaluating this expression numerically gives the dispersion relation for undamped plasma waves in the limit of vanishing amplitude. The dispersion curve is shown in Figure 1, where there are two distinct branches. The upper branch corresponds to an undamped form of the Langmuir wave and the lower branch, with $\omega < \omega_{Pe}$ corresponds to the Electron Acoustic Wave.

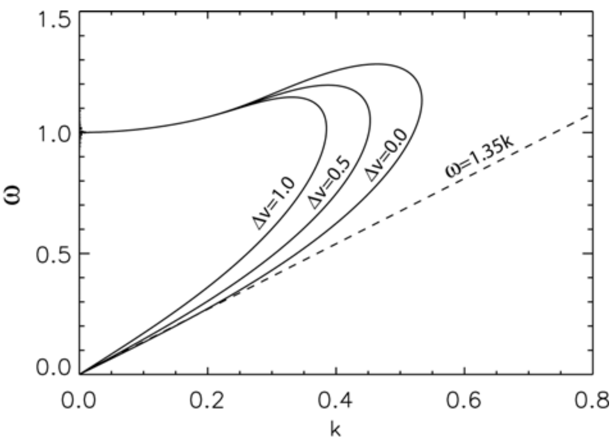

Figure 1. Dispersion relations for undamped BGK-like plasma modes in the linear limit (where the width Δv of the flattened region in units of thermal velocity, tends to zero) and for two non-linear cases (Δv=0.5, 1.0). The lower branch represents the Electron Acoustic Wave, which for low *k* follows ω= 1.35*k*. The upper branch represents an undamped form of the Langmuir mode.

Similar analysis can be carried out for distribution functions with a finite flattened region of width proportional to the wave amplitude. This gives a family of dispersion curves inside the ideal, infinitesimal amplitude case as shown in Figure 1.

Physical Interpretation

The EAW appears, at first, to be unphysical. In particular, its low frequency is a characteristic not expected of electron plasma waves in which ion dynamics play no role. Some physical understanding, for the case of small *k*, can be gained by considering a flattened distribution to be a superposition of a background Maxwellian population of electrons and a smaller drifting population, as shown in Figure 2.

In the frame of reference of the background population, oscillations at the plasma frequency in the second population will be Doppler shifted such that

$$\omega = \omega' + kv \quad (5)$$

where

$$\omega' = \sqrt{\frac{e^2 n_2}{\varepsilon_0 m_e}} \quad (6)$$

is the electron plasma frequency for the second population. In the limit where the density of this second population tends to zero we recover $\omega \sim vk$.

This dispersion relation is linear in *k* at small *k*, and implies a frequency below the plasma frequency. While this interpretation does not give the value for the phase velocity of the EAW (v_p=1.35) it is helpful in understanding the origin of low frequency electron plasma modes.

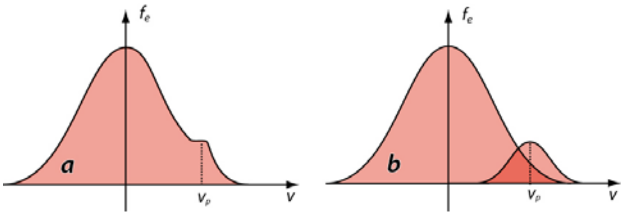

Figure 2. The flattened, trapped electron distribution (a) can be considered as a background Maxwellian plus a second electron distribution centred at v_p (b). The Doppler shifted frequency of plasma oscillations in the second electron distribution gives the EAW dispersion relation in the limit of small density.

Simulating an EAW

A full treatment of the EAW requires a kinetic description of the plasma[5]. This section outlines the fully kinetic Vlasov-Poisson model and the development of initial conditions for, and the simulation of, a travelling EAW.

The model used is a one-dimensional Vlasov-Poisson system of electrons and immobile protons with no magnetic field[6] and has been used previously to demonstrate kinetic phenomena of relevance to LPI[7]. This fully nonlinear self-consistent system is governed by the Vlasov equation for the electron distribution function f_e,

$$\frac{\partial f_e}{\partial t} + v\frac{\partial f_e}{\partial x} - \frac{e}{m_e}E\frac{\partial f_e}{\partial v} = 0 \qquad (7)$$

and Poisson's equation for the electric field

$$\frac{\partial E}{\partial x} = -\frac{e}{\varepsilon_0}\left(\int f_e dv - n_i\right) \qquad (8)$$

As an initial condition, consider an unperterbed distribution function flattened at a phase velocity v_p chosen from the EAW branch of the dispersion relation (Equation (3)). We specify

$$f_u = f_0 + f_1 \qquad (9)$$

where f_0 represents a Maxwellian distribution, and

$$f_1(v) = \partial_v f_0\big|_{v_p}(v - v_p)\exp\left(\frac{-(v-v_p)^2}{\Delta v^2}\right) \qquad (10)$$

The width of the flattened region, which relates to the quantity of trapped electrons, is given by Δv and is proportional to the EAW amplitude, since the wave is supported by trapped electrons.

In order to create a travelling wave we perturb the distribution function given in Equation (9) by adding

$$f_p(x,v) = \frac{-eE_0}{m_e(\omega - kv)}\sin(kx - \omega t)\partial_v f_u \qquad (11)$$

obtained by a linear perturbation of the Vlasov equation. Note that Equation (9) contains no singularities since $\partial_v f_u\big|_{v_p} = 0$ and ω is chosen to be real.

The Vlasov-Poisson system was initialised in a periodic box with the distribution function $f_e = f_u + f_p$ and $\omega=0.6\omega_{Pe}$, $k=0.4k_D$. In this regime we would not normally expect an electron plasma wave to propagate, undamped or otherwise. Figure 3 shows the trapped electron distribution of the EAW after a thousand inverse plasma frequencies. Figure 4 shows that the amplitude of the EAW is effectively constant after an initial transient phase, only weak numerical damping remains. A non-Maxwellian distribution, specifically the flattening at the phase velocity of the wave, is a necessity for the propagation of an EAW. This requires a kinetic model which can accurately resolve the complex phase-space structure, which is a particular strength of the Eulerian Vlasov code.

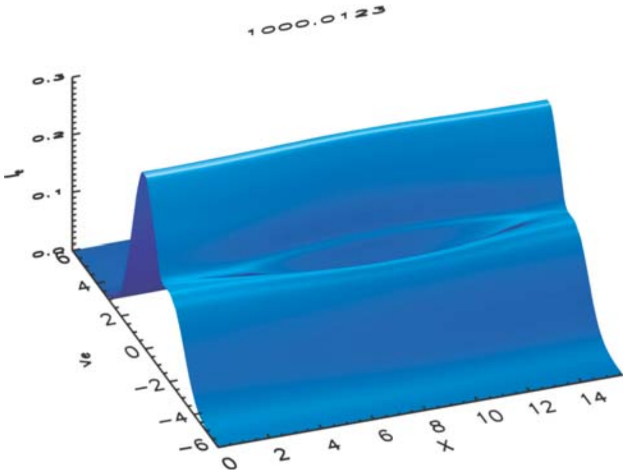

Figure 3. Electron distribution function for a large amplitude EAW $(\delta n = 0.1n_e,\ \omega = 0.6\omega_{pe},\ k = 0.4k_D)$ at time $t = 10^3\omega_{Pe}^{-1}$, simulated using the Vlasov-Possion code.

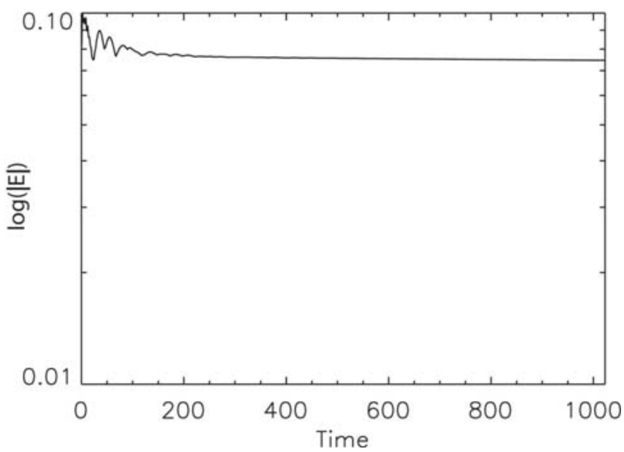

Figure 4. Logarithmic amplitude of EAW ($\delta n=0.1n_e$, $\omega=0.6\omega_{Pe}$, $k=0.4k_D$) against time. After an initial transient stage, the EAW persists as an undamped (except for limited damping due to numerical diffusion) electron plasma wave with frequency below the plasma frequency.

Stimulated Electron Acoustic Scattering (SEAS)

The phenomenon of SEAS, the collective scattering of incident laser light from an EAW, was identified experimentally by Montgomery *et al.*[1,2] in single hot-spot experiments conducted on the Trident laser system. Here we simulate SEAS using a Vlasov model extended to include the effects of transverse fields. We solve the relativistic Vlasov equation for mobile electrons

$$\frac{\partial f_e}{\partial t} + \frac{p_x}{m_e}\frac{\partial f_e}{\partial x} - \frac{e}{m_e}\left(E_x + v_y B_z\right)\frac{\partial f_e}{\partial p_x} = 0 \qquad (12)$$

together with Maxwell's equations for the transverse fields

$$\frac{\partial E_y}{\partial t} = -c^2 \frac{\partial B_z}{\partial x} - \frac{J_y}{\varepsilon_0}, \ \frac{\partial B_z}{\partial t} = -\frac{\partial E_y}{\partial x}. \qquad (13)$$

Transverse motion of particles is treated as fluid-like, hence

$$\frac{\partial v_y}{\partial t} = -\frac{e}{m_e} E_y, \ J_y = -en_e v_y \qquad (14)$$

Poisson's equation is solved as before, to give the longitudinal electric field.

The initial conditions are chosen to prohibit conventional SRS (i.e. above quarter critical density) and to satisfy wavenumber and frequency matching conditions for SEAS. Vlasov codes are inherently noiseless, so a low amplitude density perturbation is added to a Maxwellian velocity distribution to seed the growth of the EAW.

Figure 5 shows the electron distribution function at late time. The evolution of trapped electron structures, and resulting flattening of the distribution function, is visible, corresponding to an EAW. We thus have SRS-like scattering in a plasma of greater than quarter critical density from an EAW – an electron plasma wave with a frequency below the plasma frequency.

Recent work[8,9,10] has highlighted the need for a better understanding of LPI, particularly in the regimes currently being approached by the next generation of lasers. Fluid-based treatments are not alone sufficient, and the saturation of SRS via LDI, Stimulated Brillouin Scattering (SBS), SEAS and the interplay between various instabilities must be considered as a fully kinetic problem. The accurate representation and evolution of the particle distribution functions provided by a Vlasov code, therefore, make it a valuable tool. While a full 3D treatment is beyond the limits of current computing power, 1D and 2D Vlasov systems are tractable and can address many LPI problems.

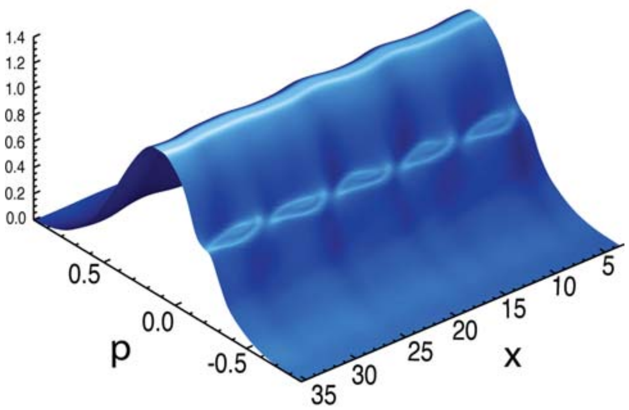

Figure 5. Surface plot of the electron distribution function at $t = 10^4 \omega_{Pe}^{-1}$. Only a small section of the complete system is shown for clarity. Electron trapping and flattening of the distribution function can be seen: this is the EAW which has grown from a background density perturbation as a result of SEAS. Axes are given in relativistic units, c/ω_{Pe} for space and $m_e c$ for momentum.

Langmuir Decay Instability

A Langmuir wave can decay into a second Langmuir wave of lower wavenumber and an ion-acoustic wave (IAW). This process can occur repeatedly forming a Langmuir cascade[11]. Can the EAW perform the role of IAW to produce a Langmuir cascade on electron timescales?

The conventional Langmuir cascade[12] proceeds for all *k* above a critical value k_c, determined by the point where the gradient of the dispersion relation of the parent Langmuir wave (L) is equal to that of the IAW:

$$\frac{\partial \omega_L}{\partial k} = \frac{\partial \omega_{IAW}}{\partial k} \Rightarrow k_c = \frac{1}{3}\sqrt{\frac{m_e}{m_i}} k_D. \qquad (15)$$

A similar analysis, for small *k*, can be performed in the case where the IAW is replaced with an EAW. Approximating the Langmuir dispersion relation by

$$\omega \approx \omega_{Pe}\left(1 + 3k^2/2k_D^2\right) \qquad (16)$$

and the EAW dispersion relation by

$$\omega \approx \omega_{Pe}(1.35k/k_D) \qquad (17)$$

gives a critical wavenumber

$$k_c \approx 0.45 k_D, \qquad (18)$$

indicating that LDI via the EAW may, indeed, be a possibility. However, Equations (16) and (17) are no longer valid for such a high critical wavenumber. The assumption of small *k* is therefore abandoned and the gradients calculated numerically to give Figure 6. Hence a full treatment, valid for all *k*, indicates that a process of Langmuir decay via the electron acoustic branch is not possible. However, this does not rule out all forms of interplay between LDI and EAWs. The upper branch of the dispersion relation, essentially an undamped form of the conventional Langmuir dispersion relation, may replace one or both of the Langmuir waves in the LDI without affecting the critical wavenumber. This problem is left for future work.

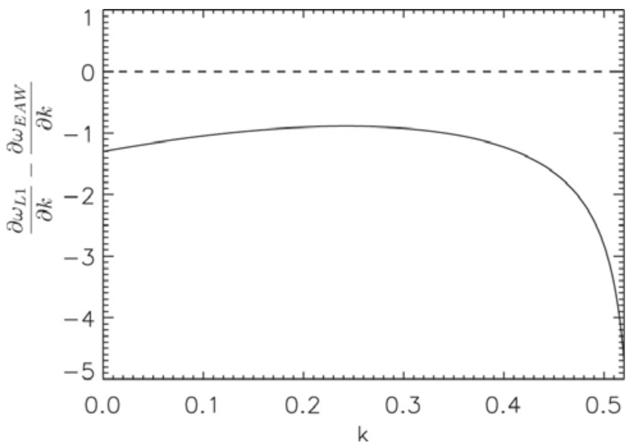

Figure 6. Difference between the gradients of the dispersion relations of the Langmuir and Electron Acoustic modes for a range of *k*. In order for Langmuir decay to occur this quantity must be greater than zero. The critical wavenumber k_c is the point at which the gradients are exactly equal. This curve remains negative for all *k*, demonstrating that Langmuir decay via the EAW, rather than the IAW, is not possible.

Summary

The Electron Acoustic Mode is a counter-intuitive phenomenon: an electron plasma wave which propagates, free from Landau damping, at frequencies below the plasma frequency. We have clarified its characteristics, in terms of dispersion relation and the role of electron trapping, and these present an interesting application of kinetic theory. Recent observations[1,2] of scattering from EAWs demonstrate the possibility for LPI involving the EAW, even in regimes where (for example) SRS is prohibited. While the expected reflectivity from processes such as SEAS may be low in present operating regimes, this work highlights the advantages of full kinetic treatments of LPI, and of the Vlasov code in particular. The Vlasov code's ability to accurately evolve the electron (and if necessary ion) distribution functions, noise-free and at high resolution over the complete phase space, ensures an accurate treatment of phenomena such as particle trapping. This is clearly of great importance to SEAS, but it is also vital to the saturation of the Raman scattering instability.

The possibility of LDI involving an EAW is an interesting one. Since it would allow a potential saturation mechanism for SRS to evolve on electron (rather than ion) time scales. However, the analysis presented here demonstrates that the IAW cannot be replaced by an EAW in this context.

References

1. D S Montgomery, J A Cobble *et al.,*
   Phys Plasmas, 9, 2311, (2002)
2. D S Montgomery, R J Focia, H A Rose *et al.,*
   PRL, 87, 155001, (2001)
3. T H Stix,
   "The Theory of Plasma Waves", McGraw Hill (1962)
4. J P Holloway and J J Dorning,
   Phys. Rev. A, 44, 3856, (1991)
5. H A Rose and D A Russell,
   Phys. Plasmas, 8, 4784, (2001)
6. T D Arber and R G L Vann,
   JCP, 180, 339, (2002)
7. N J Sircombe, T D Arber and R O Dendy,
   Phys. Plasmas, 12, 012303, (2003)
8. D Pesme, S Huller, J Myatt *et al.,*
   Plasma Physics and Controlled Fusion, 44, B53, (2002)
9. C Labaune, H Bandulet, S Depierreux *et al.,*
   Plasma Physics and Controlled Fusion, 46, B301, (2004)
10. S H Glenzer, P Arnold, G Bardsley *et al.,*
    Nuclear Fusion, 44, (2004)
11. S Depierreux, J Fuchs, C Labaune *et el.,*
    PRL, 84, 2869, (2000)
12. S G Thornhill and D ter Haar,
    Physics Reports, 43, 43, (1978)