A new diagnostic for very high magnetic fields in expanding plasmas

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It was shown in recent experiments that strong magnetic fields can be generated by intense laser plasma interaction. Magnetic fields of 700(±100) Mega-Gauss were inferred from polarization shifts of low order VUV harmonics induced by the Cotton-Mouton effect[1,2]. An alternative method, already used in laser plasma experiments, would be the direct measurement of the Faraday rotation of a given probe laser beam[3-5].

Here we describe a new diagnostic method for the magnetic field value inside the plasma, based on the idea of photon acceleration, or photon frequency shift[6]. This method was used in the past to measure the relativistic velocity of laser created ionization fronts[7,8]. But it can also be used to measure the magnetic field amplitude and the velocity of the expanding plasma boundary (or, alternatively the electron plasma density).

Let us briefly describe the basic idea of the proposed diagnostic. When the various harmonics of the incident laser field come out of the plasma, they have to cross the plasma moving boundary. It is well known that this results in a frequency shift of the harmonics by an amount that strictly depends on the velocity of the plasma front $\vec{u}$. Such a frequency shift is frequency dependent, and we expect the effect to be stronger for those harmonics with an initial frequency close to the cut-off frequencies of the different photon eigenmodes that are allowed to propagate in the medium.

The frequency shift is also polarization dependent, which means that, by comparing the observed frequency shifts for the two orthogonal polarization states, for photons propagating along a given direction with respect to the magnetic field, we will be able to measure the magnetic field amplitude, as well as the expanding velocity, if the plasma frequency is known. Notice that the harmonics are linearly polarized, if the incident laser field is also linearly polarized, but they will decompose in two orthogonally polarized modes, for instance, the right and left circularly polarized modes for propagation parallel to the magnetic field. A discussion of the practical applicability of such a diagnostic will be presented here. Examples for laser-target experiments in the Peta-Watt regime, and for radiation emitted from the polar caps of neutrons stars will be given.

The generation of nearly static magnetic fields in laser produced plasmas is well documented. Toroidal, as well as axial, magnetic fields can be created[9]. Toroidal magnetic fields are generated if the electric field inside the plasma has a non-vanishing curl: ie the electric field is not derivable from a potential. A famous example of this case is given by the non-vanishing vector product of the plasma density gradient and temperature gradient. Another possible case is the fountain-like flow of electrons, where a toroidal magnetic field is created.

Axial magnetic fields have also been created by using the inverse Faraday effect[3-5]. A circular polarized laser can induce an axial magnetic field in the plasma. This magnetic field arises because the electrons quiver with the oscillating electric field of the incoming laser light, and if the laser is circularly polarized then the electrons describe a circular motion. The net effect of this is a circular current on the edge of the plasma which generates the magnetic field. The existence of a ponderomotive force also contributes to the increase of the axial magnetic field[10,11].

In astrophysics, neutron stars and magnetized white dwarfs have extremely high magnetic fields $B \sim 10^{10} - 10^{13} Gauss$, and $B \sim 10^{6} - 10^{9} Gauss$ respectively. Magnetars have fields in excess of $10^{14} Gauss$. The fields of neutron stars are now within range of Peta-Watt laser experiments. The magnitude of the B-fields in these objects is obtained from Zeeman effect on atomic spectra, or by the observation of cyclotron absorption lines in the spectra. The later was successfully demonstrated from data taken by XMM-Newton[12] of the neutron star 1E1207.4-5209[13]. The results showed absorption features around 0.7keV. If this is taken to be the fundamental electron cyclotron frequency then the magnetic field is in the range of $6x10^{10} Gauss$, or $8x10^{10} Gauss$ taking into account a standard 20 percent gravitational red shift.

Let us consider a sharp boundary, moving with reduced velocity $\beta = u/c$, where c is the speed of light, along some given direction O*x*. It can be shown that the frequency of a photon crossing the boundary varies according to[14]

$$\omega(1 - n\beta) = \omega'(1 - n'\beta) \qquad (1)$$

where n is the plasma refractive index, and n' is the refractive index of the medium surrounding the plasma. In typical laser-plasma experiments this medium will be nearly a vacuum. Here ω is the photon frequency inside the plasma, and ω' its frequency in the outside region. This equation is valid in general conditions, and can be seen as the statement that phase of the electromagnetic wave (associated with the photon) varies continuously across the plasma boundary. It also implicitly contains the relativistic invariance of the photon phase. Here, for simplicity, we retain the geometric optics description of the electromagnetic radiation coming out of the plasma, but a full wave description would lead to the same value for the frequency shift, $\Delta\omega = \omega' - \omega$. Its exact expression is then given by

$$\Delta\omega = \omega\beta \frac{(n' - n)}{(1 - n'\beta)} \qquad (2)$$

In the following, we will consider $n' = 1$. Noting that the refractive index will depend on the polarization state and on the initial frequency, we can write, for each polarization state j

$$\Delta\omega_j = \omega\beta \frac{1 - n_j(\omega)}{(1 - \beta)} \qquad (3)$$

If the magnetic field in the plasma is nearly perpendicular to the direction of propagation (say, $\vec{B}_0 = B_0 \vec{e}_y$), we can use the well known expressions for the refractive index for the two polarization photon states, $i = o$ for the ordinary mode with electric field polarized in the *Oy*-direction, and $i = x$ for the extraordinary mode with electric field polarized in the plane *Oxz* (see for instance reference [9]). Introducing the dimensionless parameters $x = \omega/\omega_p$, $y = \omega_c/\omega_p$ and $\Delta\nu = \Delta\omega/\omega$ where $\omega_p = (4\pi e^2 n_e/m)^{1/2}$ is the electron plasma frequency, e, m, n_e being the electron charge, mass and density, we can write the relative frequency shift $\Delta\nu_j$, for both polarizations $j = o$ and $j = x$ as

$$\Delta\nu_o = \frac{\beta}{(1-\beta)}\left[1-\left(1-\frac{1}{x^2}\right)^{1/2}\right] \qquad (4)$$

and

$$\Delta\nu_x = \frac{\beta}{(1-\beta)}\left\{1-\left[1-\frac{(x^2-1)}{x^2(x^2-y^2-1)}\right]^{1/2}\right\} \qquad (5)$$

Of particular interest is the region near the cut-off frequencies, as shown below. The cut-offs are determined by

$$x_o = 1 \;,\; x_x = \frac{y}{2}+\sqrt{1+y^2/4} \qquad (6)$$

Near these values, the frequency up-shift attains its maximum value of

$$\Delta\nu_{o,x} \simeq \frac{\beta}{1-\beta} \qquad (7)$$

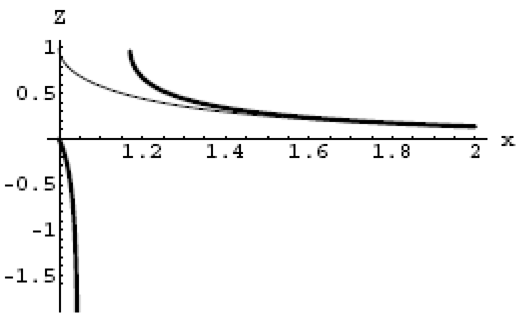

Figure 1. Frequency shifts for a perpendicular magnetic field. The figure represents the quantities $Z = (1-\beta)\Delta\nu j/\beta$, for the ordinary mode $(j = 0)$ and the extraordinary mode $(j = x)$ in bold, as a function of the normalized initial photon frequency x.

This is clearly seen in figure 1, where the frequency shifts for both modes are represented (in our dimensionless variables) as functions of the initial frequency. The up-shifts for the two modes are only equal asymptotically, at infinity. Notice that the maximum up-shifts are the same, but at different frequencies, as defined by equations (6). Notice also photons propagating in the lower branch of the extraordinary mode (below the upper-hybrid frequency $\omega_{uh} = \omega_p\sqrt{x^2+y^2)}$ will suffer a frequency down-shift which will diverge when approaching the resonance, $x_{uh} = \sqrt{1+y^2}$. This means that photons approaching the resonance in a moving plasma will be strongly down-shifted, but they will never avoid falling into the critical layer where they will finally be absorbed. Therefore, for observations outside the plasma, we can simply ignore the lower branch of the extraordinary mode.

Let us now discuss physically plausible scenarios where the above mechanism can be applied as a new plasma diagnostic. Let us consider first the dense plasma created by intense laser target interactions. The resulting magnetic field could be measured by using the harmonics of the incident laser field, with frequencies $\omega_l = l\omega_0$, that are generated inside the plasma and have to cross its moving boundary before being detected

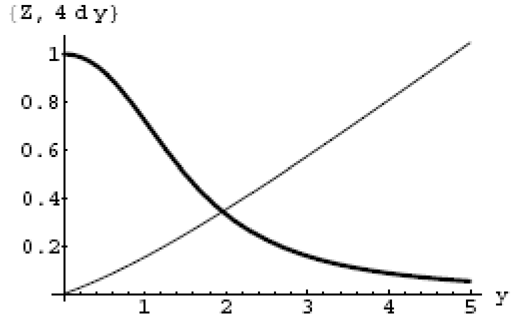

Figure 2. Frequency shifts for a perpendicular magnetic field. The figure represents the quantities $Z = (1-\beta)\Delta\nu_0/\beta$ (measured at the cut-off $x = x_x$, in bold) and $d(y) = 2y + 2(4+y^2)^{1/2}$, as functions of the normalized cyclotron frequency y.

outside. Thus, the laser harmonics can be used as natural probes in this scenario.

The plasma is expanding at a velocity that is roughly proportional to the square root of the laser intensity, $v \sim I^{1/2}$. We can then use the following empirical scaling

$$\beta = \beta_0 I_{15}^{1/2} \qquad (8)$$

Where I_{15} is the laser intensity measured in units of $10^{15} Wcm^{-2}$. Experiments and hydrodynamic simulations indicate that $\beta_0 = 3x10^{-4}$. This means that values of $\beta \sim 0.1$ should be expected for intensities as high as $I_{15} = 10^5$. Therefore, measurable frequency upshifts of nearly ten percent can be observed for some harmonic frequency ω_l that is emitted nearly at cut-off: $(\Delta\omega_l)\max \simeq \beta$, for $\beta << 1$. We can then see that, by identifying the extraordinary cut-off and by measuring the frequency shifts of both modes around this position, we will be able to measure β, if the plasma frequency is known by some other method. So, we could get a simultaneous diagnostic of the magnetic field amplitude and of the expanding plasma velocity.

A similar discussion could also be made for the case of parallel magnetic fields[6)], even if this case is less likely to occur for the total B-field associated with intense laser target interactions. In contrast, this case would be ideal for radiation emitted by the polar regions of a neutron star, in the presence of relativistic plasma jets.

In conclusion, we have discussed a new method of diagnostic for intense magnetic in expanding plasmas, which can be used to study dense plasmas generated by laser target interactions in the PetaWatt regime and in astrophysical objects. This method is based on the frequency shift of radiation crossing the moving boundary of the expanding plasma. The harmonics of the incident laser field could be used as natural probes of the plasma medium. The frequency shift is polarization dependent and the observation of the shifted both polarization states would also allow the measurement of the velocity of the plasma expansion. It was shown that magnetic fields in the range of $10^9 Gauss$ could easily be studied.

This concept can easily be tested using PetaWatt laser systems and hopefully will lead to a new technique to measure magnetic field intensities in both the laboratory and astrophysics.

References

1. U Wagner *et al.*,
   Phys. Rev. E, 70, 026401 (2004)
2. M Tatarakis *et al.*,
   Nature, bf 415, 280 (2002);
   M Tatarakis *et al.*,
   Phys. Plasmas, 9, 2244 (2002)
3. Z Najmudin *et al.*,
   Phys. Rev. Lett., 87, 215004 (2001)
4. M Borghesi *et al.*,
   Phys. Rev. Lett., 81, 112 (1998)
5. Y Horovitz *et al.*,
   Phys. Rev. Lett., 78, 1707 (1997);
   Y Horovitz *et al.*,
   Phys. Lett. A, 246, 329 (1998)
6. S Eliezer, J T Mendonca, R Bingham and P Norreys,
   Phys. Lett. A, 336, 390 (2005)
7. J M Dias *et al.*,
   Phys. Rev, Lett., 78, 4773 (1997)
8. N Lopes *et al.*,
   Europhys. Lett., 66, 371 (2004)
9. S Eliezer,
   The Interaction of High-Power Lasers with Plasmas,
   Institute of Physics Publishing, Bristol (2002)
10. T Lehner,
    Phys. Scripta, 49, 704 (1994)
11. Z.M. Sheng and J. Meyer-ter-Vehn,
    Phys. Rev. E, 54, 1833 (1996)
12. G F Bignami *et al.*,
    Nature, 423, 725 (2003)
13. B J Kellett *et al.*,
    MNRAS, 225, 199 (1987)
14. J T Mendonca,
    Theory of Photon Acceleration, Instutite of Physics
    Publishing, Bristol (2001)