# Analysis on a Wedge-shaped Thomson spectrometer for ion studies 

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## Introduction

The Thomson Spectrometer is an important plasma diagnostic instrument used for detecting the nature and energy distribution of different ion species. It employs static, parallel magnetic and electric fields acting transversely on the direction of motion of the ions initially incident along the device, defined by an input pinhole in front of the instrument. The forces due to the electric and magnetic fields on a charged particle result in a deflection of its path in transverse plane. For a distribution of energies among the same or, different non-relativistic ions of identical charge-to-mass ratio, the resulting trace (known as dispersion curve) on a transverse detector plane forms a parabola and hence the device is called Thomson Parabola spectrometer (named after J.J. Thomson, who invented the mass spectrometer, which relies on the same principle).

A conventional Thomson parabola includes a pair of parallel permanent magnets and a pair of parallel high voltage electric plates, but the present article will analyse a modified Thomson Parabola developed at the Central Laser Facility that includes a wedged shaped electric plate pair to increase deflection of ions on detector film (CR39). Figure 1 describes the schematic design of the modified parabola. For the present analysis, the magnetic field is taken constant and the variation of electric field is modeled analytically.


Figure 1. Schematic of the modified Thomson parabola spectrometer designed in CLF, RAL.

## Mathematical analysis: simulation of charged particle trajectory

The equation of motion of a charged particle of charge ' $q$ ' and mass ' $m$ ' inside the device is given as follows (see also appendix I):

$$
\begin{equation*}
x=v_{0} t ; \quad \ddot{z}=\frac{q}{m} v_{0} B ; \quad \ddot{y}=q E(x) \tag{1}
\end{equation*}
$$

where $v_{0}$ is the initial velocity of the charged particle along longitudinal ( x ) direction; B is 0.6 T , the average value for the experimentally measured magnetic field and $\mathrm{E}(\mathrm{x})$ is the electric field in y-direction. Electric field is variable due to wedge shape of the gap between two plates. If ' V ' is the voltage applied between two plates whose gap varies from $\left(h_{1}=0.002 \mathrm{~m}\right)$ to $h_{2}(=0.022 .5 \mathrm{~m})$ over a length L
( $=200 \mathrm{~mm}$ ), the electric field can be expressed as:

$$
\begin{equation*}
E(x) \cong V / y=V / h_{1}+\frac{\Delta h}{L} x \tag{2}
\end{equation*}
$$

where $\Delta h=h_{2}-h_{1}=0.0205 \mathrm{~m}$.
The origin $(0,0)$ in $y-z$ plane is referred to the pinhole position.

It is reasonable to assume that the initial conditions are:
$\underbrace{\dot{x}=v_{0}, \ddot{x}=0 ; \quad y, \dot{y}, \ddot{y}=0 ; \quad z, \dot{z}, \ddot{z}=0}_{t=0}$
It can be shown using equations (1) to (3) that under the influence of electric and magnetic fields the charged particle will follow the curved trajectory given by
$z=\frac{1}{2}\left(\frac{q}{m}\right) B \frac{1}{v_{0}} x^{2}$
$y=$
$\left(\frac{q}{m}\right) \frac{V L}{\Delta h v_{0}^{2}}\left[x \ln \left(1+\frac{\Delta h}{L h_{1}} x\right)-x+\frac{h_{1} L}{\Delta h} \ln \left(1+\frac{\Delta h}{L h_{1}} x\right)\right]$
The first term in equation (5) is the most dominant one. Further, if it is assumed that the fields (magnetic or electric field) do not extend beyond the extents of their physical sources, i.e., the length of magnet $(0.05 \mathrm{~m})$ or the electric plate $(0.2 \mathrm{~m})$; the particle trajectory can be described by the tangents to the curved trajectories at the positions of the physical ends of the sources. Finally, the co-ordinates of the charged particle on the detector (CR39 film) plane, which is $0.2 \mathrm{~m}(0.05 \mathrm{~m})$ away from the end of magnet (electric plate); can be written as:
$z=$
$\left.\frac{1}{2}\left(\frac{q}{m}\right) B \frac{1}{v_{0}} x^{2}\right|_{x=0.05}+\left.0.2 \frac{d}{d x}\left(\frac{1}{2}\left(\frac{q}{m}\right) B \frac{1}{v_{0}} x^{2}\right]\right|_{x=0.05}$
$y=$
$\left.\left(\frac{q}{m}\right) \frac{V L}{\Delta h v_{0}^{2}}\left[x \ln \left(1+\frac{\Delta h}{L h_{1}} x\right)-x+\frac{h_{1} L}{\Delta h} \ln \left(1+\frac{\Delta h}{L h_{1}} x\right)\right]\right|_{x=0.2}+$ $\left.0.05 \frac{d}{d x}\left[x \ln \left(1+\frac{\Delta h}{L h_{1}} x\right)-x+\frac{h_{1} L}{\Delta h} \ln \left(1+\frac{\Delta h}{L h_{1}} x\right)\right]\right|_{x=0.2}$

After simplification, using $\mathrm{V}=4000 \mathrm{~V}$ and $\mathrm{B}=0.6 \mathrm{~T}$, the following results are obtained:
$z=(0.00075+0.006)\left(\frac{q}{m}\right) \frac{1}{v_{0}}=0.00675\left(\frac{q}{m}\right) \frac{1}{v_{0}}$
$y=(12928.79+4722.67)\left(\frac{q}{m}\right) \frac{1}{v_{0}^{2}}=17651.46\left(\frac{q}{m}\right) \frac{1}{v_{0}^{2}}(9)$
Tracks for various charged particles were simulated for velocity ranges assuming a Maxwellian distribution of energies. Figure 2 represents a simulation of tracks for protons, $\alpha$-particles, carbon ions ( +6 and +1 ) and neutral particles for a Maxwellian energy distribution at 100 MeV mean temperature. Carbon and Hydrogen were chosen for common presence in target contaminants. In practice, the origin (i.e. where the neutral particles hit the detector plane) of local co-ordinate system on detector is not at $(0,0)$. The shift in origin $(15,15)$ is made to match the experimental data discussed later in the paper.


Figure 2. Simulation for tracks for proton, $\alpha$-particle and carbon ions ( +1 and +6 ) for the modified Thomson spectrometer using a constant magnetic field of 0.6 T and a dc electric voltage of 4 KV .

Using equations (8-9), the location of each individual charged particle, with a specific charge-to-mass ratio, on the detector plane was mapped using various inputs for energy. Such dispersion data (see Figure 3) is useful to predict the energy of the ions. It is easy to see that in the $y-z$ plane a horizontal line represents a contour of equal momentum per charge, while a vertical line represents a contour of equal energy per charge. Any line passing through the origin represents a constant velocity line. ${ }^{(1)}$


Figure 3. Simulated dispersion data for proton, $\alpha$-particle and carbon ions $(+1$ and +6$)$ for various energy values at $V=4 \mathrm{KV}$ and $B=0.6 \mathrm{~T}$.

## Inverse transform

The equations (8-9) can be solved for $\frac{q}{m}$ and $v_{0}$ analytically as follows:
$q_{m}=4.047 \frac{(z \pm \Delta z)^{2}}{(y \pm \Delta y)} ; v_{0}=2615832.55 \frac{(z \pm \Delta z)}{(y \pm \Delta y)}$
where $\Delta y$ and $\Delta z$ are uncertainties in co-ordinates of origin. It may be noted that in equation (10), $q_{m}$ is normalised with respect to the proton charge-to-mass ratio ( $=95788309 \mathrm{C} \mathrm{kg}^{-1}$ ) which makes $q_{m}=1$ for proton, $q_{m}=0.5$ for $\mathrm{C}^{+6}$ and $\alpha$, and $q_{m}=0.83$ for $\mathrm{C}^{+1}$. Equation (10) can be used to find the charge-to-mass ratio of unknown ion species where y and z are the raw experimental data obtained by scanning ion tracks. When

shift in origin $(\Delta y, \Delta z) \ln m m$


Figure 4. Sensitivity of the inversion data on the error in origin position (in $\mathbf{m m}$ ) simulated for three types of charged particles: proton $\left(\mathrm{H}^{+}\right), \mathrm{C}^{+6}$ and $\mathrm{C}^{+1}$
the origin is set correctly (i.e., $\Delta y=\Delta z=0) q_{m}$ vs. $v_{0}$ plot generates a set of horizontal lines (parallel to $v_{0}$ axis). This is because the experimental data contains a few specific ions (a set of specific values of $q_{m}$ ) with large variation of velocities. Plots are sensitive to the actual position of the origin of the co-ordinate system. The uncertainty in the origin can produce finite slopes and non-linearity in $q_{m}$ vs. $v_{0}$ plot. Figure 4 illustrates the sensitivity of the plots to the shift $(\Delta y, \Delta z)$ of the origin from the actual position for simulated data for proton and carbon ions.

## Experimental result

The wedged Thomson Spectrometer was employed as a proton/ion detection diagnostic during the Jan-Feb'06 experimental campaign at the VULCAN laser Target Area West at Rutherford Appleton Laboratory. The Thomson spectrometer was placed 30 degrees off the vertical axis of the target chamber. Since the emission of ions was weak in that direction, a larger pinhole was ( $600 \mu \mathrm{~m}$ diameter) used to increase the collection of ions. The resolution ${ }^{[1]}$ is calculated to be about $765 \mu \mathrm{~m}$ on a detector plane 285 behind the pinhole for a 10 mm size source situated at a distance of 1050 mm in front of the pinhole.

The experiment employed short ( $\sim 1 \mathrm{ps}$ ), dual chirped pulse amplified (CPA) beams from the VULCAN laser operating at $1.054 \mu \mathrm{~m}$ wavelength. One beam was used to create protons from a probe target while a second beam was used to illuminate the sample target. During the experimental campaign, $100 \mathrm{mg} / \mathrm{cc} \mathrm{CHO}$ foams mounted in Al half washers were used as sample targets and $25 \mu \mathrm{~m}$ Au foils were used for probe targets. Typical energies were 50 J for the interaction beam and 15 J for the probe beam. Figure 5 represents typical data for ion tracks on CR39 film for a 4 KV dc bias Thompson spectrometer. The data for the Figure 5 was generated from scanning a NaOH etched CR39 film using a TASLIMAGE ${ }^{\mathrm{TM}}$ scanning system from Track Analysis Systems Ltd. The multi parabola tracks belong to the ions of Hydrogen, Carbon, Oxygen, Gold, and probably the Aluminum, as these are the elements present as target components. The simulation


Figure 5. Scanned image of a $50 \mathrm{~mm} \times 50 \mathrm{~mm} \times 1 \mathrm{~mm}$ CR39 film. The useful data is marked within a drawn blue window showing: different ions tracks; images of two distinct sources, and a horizontal streak of secondary scattered ions generated in space beyond the magnetic field influence but within the electric field influence.
data from Figure 3 suggests that peak energy in $\mathrm{H}^{+}$, and $\mathrm{C}^{+1}$ ions are less than 5 MeV , while the peak energy of $\mathrm{C}^{+6}$ is $\sim 10 \mathrm{MeV}$. The neutral atoms from the two sources (interaction and probe targets) form two images of the sources at the tracks origin $(15,15)$. One interesting feature of the Figure 5 is the presence of a horizontal linear track passing through the origin. This could be attributed to the generation of secondary ions from the neutral atoms ${ }^{[2]}$ beyond the magnetic field influence but still inside the electric field influence. This is possible as the magnet length is 50 mm , while the electric field plate is 200 mm long.

## Conclusion

A wedge shaped Thomson Spectrometer developed at CLF was analysed. The tracks and energy dispersion of ions was simulated analytically. A novel method of retrieving ion charge-to-mass ratio has been suggested. However, further work is necessary to apply the method in presence of noisy data. The device was used to collect experimental data from an experiment using the VULCAN laser facility.

## Appendix I

The general set of equations, which accurately describe the motion of charge particles under the action of parallel Electric and magnetic fields, will be:
$m \frac{\partial}{\partial t}\left(v_{x}\right)=-q v_{z} B ; m \frac{\partial}{\partial t}\left(v_{z}\right)=q v_{x} B ; m \frac{\partial}{\partial t}\left(v_{y}\right)=q v_{y} E$

The coupled differential equations in $x$-and $z$-velocity components result in a circular trajectory of the charged particle in the $\mathrm{x}-\mathrm{z}$ plane around the B -direction (y). Moreover, particles with different energies will arrive at different times on the detector due to varying dragging force along axial direction.

In practice, the magnitude of fastest $v_{z}$ is one order of magnitude lower than $v_{x}$ :
$v_{x} / v_{z} \approx$
$\frac{\text { Axial distance of CR39 plane from pinhole }(285 \mathrm{~mm})}{\text { Lateral distance of CR39 edge from pinhole }(35 \mathrm{~mm})} \gg 1$
Under such conditions the effect of axial (x) acceleration due to transverse $(\mathrm{z})$ velocity can be neglected and the particle can be assumed to move with uniform initial axial velocity. Such approximation does not affect the higher energy particles that spend less time in the magnetic field, but can have small effect on the lower energy particles.

Moreover, Equation (2) is valid under the approximation that the curvilinear electric field lines (along circular arcs) between the inclined electric plates can be represented by linear electric field lines (along y-direction). Such an approximation is valid for the small angle $\left(5.7^{\circ}\right)$ between the plates. Physically, it neglects any marginal axial dragging force on ions due to electric field. It may be added that the fractional electrical energy (e.g., $4 \sin \left[5.7^{\circ}\right]$ $\mathrm{KeV} \sim 400 \mathrm{eV}$ for a proton) spent up in axial direction is much less than the kinetic energy ( $\sim$ several MeV ) of the charged particles. Since as such there is no evidence of bremsstrahlung emission, the ions will not change their energy during such process.


Figure 6. Simulated Fitting tracks for various ions to the experimental data using a program developed by F. Lindau.

Figure 6 represents simulated traces fitted to experimental data of Figure 5 using a program developed by Lindau ${ }^{[3]}$ that takes into account the general velocity equations. The results are in good agreement with the predictions made by the numerical analysis present in this paper.

## References

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