Ion heating due to ionisation and recombination

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Introduction

It is very common in plasma physics, particularly in plasmas of moderately high atomic number for the average charge state $<Z> = n_e/n_i$ to be somewhat less than the nuclear charge Z_{nuc}. For instance Neon is only fully ionised for temperatures of more than 500eV or so and Xenon is only fully ionised above around 50keV. It is usual to distinguish between Z_{nuc} and $<Z>$ in the description of the plasma but much less common [1,2] to recognise that plasmas contain ions of many different charge states which each have different dynamics and collision frequencies.

In reality any one ion will not have a constant charge state but its charge will fluctuate as a result of ionisation and recombination events and each charge state will have slightly different dynamics in the plasma electromagnetic fields. In many ways the charge fluctuations are analogous to the electron-ion collisions that result in inverse bremsstrahlung absorption of electromagnetic wave energy by electrons. If the electrons are initially hotter than the ions or there is an external energy source driving plasma waves then it is possible for the ions to steadily gain energy as a result of the random changes of charge state.

Somewhat similar effects occur in dusty plasmas [3] where the electric charge on dust particles fluctuates, energy is not always conserved since the Hamiltonian $H = p^2/2M + Ze\Phi$ is time dependent through the fluctuations of Z.

Motion in an Ion-acoustic wave

We consider an idealised case where the plasma ions of mass M may exist in two charge states Z_1 and Z_2 ($Z_2 > Z_1$) and for simplicity we consider that the ionisation rate ν_{ion} from Z_1 to Z_2 is the same as the recombination rate ν_{rec} from Z_2 to Z_1 so that the equilibrium densities of Z_1 and Z_2 are equal. We also define $\nu = \nu_{ion}/2 = \nu_{rec}/2$ which is the rate for a complete cycle of ionisation and recombination.

Consider the motion of the ion in charge state Z_1 in a monochromatic ion acoustic wave described by its longitudinal electric field $E = E_0 \cos(\omega t - kx)$. Since the wave is longitudinal and assumed to be small amplitude then the density fluctuation is $(\delta n/n) = v_o/v_{ph}$ where $v_o = Z_1eE_0/M\omega$ is the oscillation velocity and $v_{ph} = \omega/k = (Z_1k_BT_e/M)^{1/2}$ is the ion acoustic speed, k_B being Boltzmann's constant. Initially the ion oscillation is around the point x = 0 and has no net drift so that

$$v = (Z_1eE_0/M\omega) \sin(\omega t).$$

We now consider what happens when the ion of charge state Z_1 is ionised to charge state Z_2 at some phase ϕ_1 of the electric field. Ionisation generally occurs either via electron impact or by photoionisation and in either case there is little change in the ion momentum. So immediately after the ionisation event we may write the ion velocity as

$$v = (Z_2eE_0/M\omega) \sin \omega t + (Z_1 - Z_2)(eE_0/M\omega) \sin \phi_1$$

The first term corresponds to the oscillatory motion in the new charge state and the second to an average drift velocity. If the ion of charge Z_2 now recombines with an electron at phase ϕ_2 to produce an ion of charge Z_1 again with little change in the ion momentum then the drift velocity after one ionisation - recombination cycle is

$$v_D = (Z_1 - Z_2)(eE_o/M\omega)\sin\phi_1 + (Z_2-Z_1)(eE/M\omega) \sin \phi_2$$

The velocity increments do not cancel out since they occur at different phases of the wave. We rewrite this last equation as

$$v_D = 2\left(\frac{Z_1 - Z_2}{Z_1}\right) v_o \cos(\phi)\sin(\frac{\delta\phi}{2}) \qquad (1)$$

where $\phi = (\phi_1 + \phi_2)/2$ and $\delta\phi = \phi_1 - \phi_2$

After many uncorrelated cycles of this process the drift velocity will perform a random walk so that the expectation value of the energy increases linearly with time.

$$\left\langle v_D{}^2 \right\rangle = 4\nu t\left(\frac{Z_2 - Z_1}{Z_1}\right)^2 v_o{}^2\left\langle\cos^2\phi\right\rangle\left\langle\sin^2\left(\frac{\delta\phi}{2}\right)\right\rangle$$

The absolute phase ϕ is random so $<\cos^2\phi> = 1/2$ and we evaluate the term in $\delta\phi = \omega\tau$ by taking the time delay τ between ionisation and recombination to be exponentially distributed so that we need to evaluate

$$\int_0^\infty \nu \exp(-\nu\tau)\sin^2\left(\frac{\omega\tau}{2}\right)d\tau = \frac{1}{2}\left(\frac{\omega^2}{\nu^2 + \omega^2}\right)$$

giving our final result for the rate of increase of ion energy

$$\left\langle\frac{1}{2}Mv_D{}^2\right\rangle = \nu t\left(\frac{Z_1 - Z_2}{Z_1}\right)^2 \frac{1}{2}Mv_o{}^2\frac{\omega^2}{\nu^2 + \omega^2} \qquad (2)$$

For small values of ν/ω the heating rate is proportional to ν while for $\nu >> \omega$ the heating rate falls as ν^{-1} since the successive ionisation and recombination events are strongly correlated. This calculation is strictly valid only for small ion wave amplitudes $(v_o/v_{ph}) = (\delta n/n) << 1$ since we assume that the ionisation and recombination rates are independent of the phase of the waves [4].

Motion In a Static Potential Well

We consider the motion of an ion initially of charge Z_1 in a potential well $e\Phi = ax^2$ with $x = x_1 \sin\omega t$ and initial energy $E_1 = Z_1ax_1{}^2$. If the ion changes its charge state to Z_2 at phase ϕ_1 with no change in momentum then the new energy is $E_2 = E_1 + (Z_2 - Z_1)ax_1^2\sin^2\phi_1$. The frequency and phase of the oscillation in the new charge state are different to their previous values and the exact algebra becomes lengthy. If we restrict the analysis to ionisation and recombination rates smaller than the oscillation period we can assume all phases to be randomly distributed and neglect the phase change at ionisation and recombination. If the ion now returns to charge Z_1 at phase ϕ_2 of its new oscillation and its new energy is E_3, then we find

$$\frac{E_3}{E_1} = \cos^2\phi_1\cos^2\phi_2 + \sin^2\phi_1\sin^2\phi_2 + \frac{Z_2}{Z_1}\sin^2\phi_1\cos^2\phi_2 + \frac{Z_1}{Z_2}\cos^2\phi_1\sin^2\phi_2$$

so that with randomly distributed ϕ_1 and ϕ_2:

$$\left\langle\frac{E_3}{E_1}\right\rangle = \frac{1}{2} + \frac{1}{4}\left(\frac{Z_1}{Z_2} + \frac{Z_2}{Z_1}\right) > 1$$

This is an exponential increase of the mean energy with each ionisation recombination cycle. It is possible because the motion of a particle with time dependent charge corresponds to a time dependent Hamiltonian and energy is in general not conserved. Truly electrostatic structures in plasmas are rare but the late stage evolution of many plasma instabilities such as the two stream instability gives rise to long lived density structures. In most cases the density structures correspond to the expulsion of electrons by the ponderomotive force associated with wave energy and so do not provide confining potentials for the ions. Quasi resonant particles in a wave see an almost static potential and will experience this exponential growth in energy until they are shifted away from resonance.

Electron Ion Equilibration

Normally electron ion temperature equilibration proceeds via the mechanism of screened binary collisions giving the Spitzer rate for $T_e >> T_i$

$$\nu_{ie} = 3.2\times10^{-9} Z^2 \ln\Lambda n_e T_e^{-3/2} \mu^{-1}$$

where $\mu = M/m_p$ and m_p is the proton mass

The process is slower than other collision rates because the large electron ion mass ratio gives only a small energy transfer in each collision.

If $ZT_e >> T_i$ the thermal plasma will have an equilibrium excitation of ion-acoustic waves with dispersion relation $\omega_{ia} = kc_s$. The energy in the ion-acoustic modes is calculated by a procedure analogous to the simple derivation of the ratio of kinetic to electrostatic energy in the Langmuir oscillation of electrons.

If the plasma is considered to be in a box of side L then modes exist for $k = 2\pi N/L$ and the maximum value of k is given by the Landau damping limit $kc_s = \omega_{pi}$ where $\omega_{pi}^2 = 4\pi n_i Z^2 e^2/M$. The energy driving the ion waves is due to the electron thermal energy so we now ascribe an energy of $k_B T_e$ to each ion mode giving after a little manipulation exactly the same result as for the Langmuir modes:

(Electron Kinetic Energy / Ion Acoustic wave energy) = $n_e\lambda_D^3$ where λ_D is the electron Debye length

It is only when plasmas become non-ideal (few particles per Debye sphere) that there is significant energy in the electrostatic modes. To use this result with (2) we need an expression for the average oscillatory velocity of the ion-acoustic modes and we note that for a mode of wavenumber k, $v_{osc}^2 \sim k^{-2}$ while the density of modes $\sim k^2$ so that the mean square oscillation velocity is

$$M\langle v_{osc}^2\rangle = Zk_BT_e / (n_e\lambda_D^3) \text{ or } \langle v_{osc}^2\rangle / c_s^2 = 1 / (n_e\lambda_D^3)$$

Using very approximate ionisation rates [5] we find that for strongly coupled plasmas of high Z, around solid density and at temperatures of hundreds of eV the ionisation heating can be important but it is in a regime where the Spitzer theory is already of limited applicability.

Conclusion

The ion dynamics driven by stochastic changes of ion charge lead to a novel ion heating mechanism which may need to be included in the analysis of some high Z plasma experiments. The simple model presented here would need to be extended for very large amplitude waves where ionisation and recombination rates may change with the density (and in some cases temperature) changes associated with the waves. Also a more general treatment would allow for a wider distribution of simultaneous ion charge states.

References

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