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Challenges in controlling matter and light

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The intense international activity involved in probing the structure of matter on all scales, with particle beams and radiation, owes much to recent advances in accelerator science and technology. Developments in the production of high power laser radiation also offer new avenues for accelerator design and new diagnostic tools of relevance to medical science, engineering and the communications industry. Contemporary issues in novel accelerator development have motivated considerable interest in the interaction between charged particles and the electromagnetic field in domains where relativistic effects cannot be ignored. Our recent research spans numerous topics ranging from fundamental issues associated with radiation reaction, to the development of efficient analytical methods for obtaining electromagnetic fields driven by relativistic particle beams. A recurring theme in our work is the examination of *effective* theories of large collections of electrically charged relativistic particles and their electromagnetic couplings.

The following is a brief account of recent work emphasising certain challenges associated with the interaction between matter and electromagnetic fields. The background literature associated this subject is vast and no attempt is made here to provide a complete list of references.

Any self-consistent theory describing a large collection of charged particles must include all electromagnetic forces between the particles. However, the notorious problem of determining the classical force on a single accelerating point charge due to its *own* electromagnetic field has stimulated research for over a century and remains unresolved (see [1,2] for recent reviews). The structure of an isolated single electron is currently beyond observation and one often proceeds classically by associating the electron with a singularity in the electromagnetic field described by Maxwell's equations in vacuo. Following Dirac [3], an equation of motion for the charged particle may be obtained by appealing to conservation of the total energy-momentum of the particle and its electromagnetic field. In order to remove singularities in the equation of motion Dirac made "natural assumptions" about the origin of the electron mass. The resulting Lorentz-Dirac equation of motion contains the acceleration of the particle and its proper time derivative and possesses solutions that violate intuition. In particular, unless special conditions are adopted for the final state of the electron, it predicts that a free electron in vacuo can self-accelerate; furthermore the equations predict solutions where the electron may experience a sudden acceleration before it enters a region of space containing a non-vanishing external electrostatic field.

Although one can employ approximation schemes that circumvent some of the above difficulties [4], it is not clear that such methods are applicable to distributions of relativistic particles with sufficiently high proper number density. As schemes for accelerating charged particles become more complex and ambitious in their aims such approaches may be inadequate for a proper understanding of new challenges.

If one forgoes the use of point particle methods in favour of self-consistent effective theories based on classical continuum methods [1,5], the existence of non-linearities in the governing partial differential transport equations gives rise to other challenges. In particular, the matter density in relativistic simple (single-component) fluid models of a beam of charged particles may exhibit singular behaviour [5] due to the evolution of shocks. This situation is analogous to shock formation in fluid and gas dynamics. Similarly, the electron velocity field of a cold plasma undergoing sufficiently large amplitude electrostatic oscillations may become multi-valued [6]. However, the dominant inter-particle forces in a *cold collisionless* plasma are long-range and multiple streams can form dynamically; in particular, particles may become trapped in an electrostatic wave and fine-scale mixing may destroy the wave. The evolution of an electrically charged continuum with a dynamical number of components was studied in [5].

The maximum sustainable amplitude (the "wave-breaking limit") of non-linear electrostatic oscillations has been a subject of considerable interest for over half a century [6–11]. Recent years have seen a resurgence of interest in the wave-breaking limit of *warm* plasma oscillations based on macroscopic fluid (hydrodynamic) models of plasmas [8]. It has been noted that the wave-breaking limit is highly sensitive to the details of the macroscopic fluid model [9].

Plasmas dominated by collisions can be close to thermodynamic equilibrium. However, an intense and ultrashort laser pulse propagating through an under-dense plasma will drive the plasma anisotropically over a plasma oscillation period, and the plasma is effectively collisionless over such timescales. Macroscopic fluid quantities, such as number density and pressure, are induced from appropriate moments of phase-space distributions. To describe the dynamics of a collisionless plasma one employs the collisionless Vlasov equation to evolve the distribution forward in time. A programme of work was initiated recently to investigate the influence of the initial electron distribution on the maximum electric field strength [10,11]. Our approach employs a self-consistent 3-dimensional generalization of the 1-dimensional relativistic waterbag model developed by Katsouleas and Mori [7], and is part of an on-going quest for a better analytical understanding of wave-breaking in warm plasmas.

Macroscopic fluid models are particular examples of theories based on the relativistic continuum paradigm. The vanishing of the space-time divergence of the total stress-energy-momentum tensor (describing matter and electromagnetic fields) leads to relativistic equations of motion and continuity equations for the material continuum. A specification of the total stress-energy-momentum tensor defines the (constitutive) model of the fully interacting continuum including fields and matter. Macroscopic descriptions of continua may involve *effective* theories demanding substantial input from experiment. Before the advent of precision experimentation and modern gauge descriptions of the the interaction of light with matter, stress-energy-momentum tensors associated with electromagnetic fields in a classical medium were guessed on the basis of relativistic covariance. A long running dispute based on alternative proposals by Minkowski [12] and Abraham [13] about one hundred years ago has yet to be settled by experiment. Our research has shown [14] how the choice made by Abraham follows naturally from a variational principal involving gravitation and the motion of the medium, and that the symmetrized version of the choice made by Minkowski follows by alternative gravitational interactions [14,15]. Although recent experiments employing sophisticated methods based on cold atom optics seem to favour Minkowski's choice other experiments seem to prefer the Abraham tensor and a final consensus is still to be achieved [16]. These issues have led us to develop a new approach to the calculation of electromagnetic forces and torques on magneto-electric media using the concept of a *drive form* [17]. This work may have relevance to the development of new accelerating structures based on materials with novel constitutive properties (meta-materials).

Beam pipes that spatially curve and taper through magnet arrays in small-gap undulators feature in the designs of advanced machines for producing pulsed sources of intense focused electromagnetic radiation. The production of femto-second radiation pulses requires high peak electric currents and the maintenance of low emittance electron beams. Extreme design criteria are required to sustain beam stability in the presence of radiation backreaction on accelerated sources in SASE X-ray sources. A direct analytic approach to this electrodynamic problem via the coupled system of Maxwell's field equations and the equations of motion for the particle beams encounters difficult problems due to nonlinearities and retardation effects. Furthermore, the geometry of the beam pipe may be such that direct numerical solution of Maxwell's equations is too inefficient, especially when sweeping across a wide range of design parameters. Such considerations motivated the development of perturbative methods for calculating the electromagnetic fields inside curved [18] and tapered [19] beam pipes. In [18] a perturbative expansion in a small parameter characterising the curvature of the beam pipe is used to determine the longitudinal wake potential inside the pipe. Longitudinal wake potentials and impedance formulae are developed in [19] for a beam pipe whose circular cross-section slowly varies with radius; there, an asymptotic approximation is developed for the electromagnetic field based on a parameter that characterizes the slow variation of the cross-section's radius. Agreement with direct numerical solution of Maxwell's equations is impressive for tapers whose geometries are representative of the next generation of lepton colliders, with narrow bunches whose lengths are a couple of millimetres. The difficult challenge remains to develop analytical methods for tackling considerably shorter bunches.

In a further attempt to address the radiation reaction problem and the breakdown of traditional Maxwellian electrodynamics in the context of high-field laser interactions with matter, attention is currently focussed on high-field nonlinear vacuum electrodynamics. One expects classical vacuum Maxwellian electrodynamics to break down near critical electric field strengths of 1.3×10^{16} V/cm or magnetic field strengths of 4.4×10^{13} G, where electron-positron pair creation becomes possible. Such fields might be reached in a future laser with peak intensity 2.3×10^{29} W/cm^2. However there may be a classical breakdown of Maxwell equations well before one needs quantum-electrodynamics. A classical fluid model of non-linear electrodynamic interaction with matter in vacuo based on the vanishing of the divergence of the total stress-energy-momentum tensor has been developed that reduces to Maxwellian electrodynamics for suitably low-field strengths. In particular, the remarkable properties of the Born-Infeld theory offer a promising constitutive model since this preserves electromagnetic duality and possesses singular charged (Bionic) vacuum solutions with finite field-strengths at the location of the singularity. Effort is currently underway to explore the properties of charged fluid flow in external fields in this theory.

In summary, although Maxwell's equations have been known for over a century and a quarter, a number of intriguing theoretical challenges are revealed when one attempts to develop effective theories and approximations inspired by the on-going advances in plasma physics, laser technology and particle accelerator science.

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