Coherent X-rays, relativistic absorption and electron transport by the vector potential

Introduction
Reflection and absorption of light have intrigued people since antiquity. Snell's law of refraction (first accurately described by Ibn Sahl in the 10th century) revealed the connection between the angles of reflection and refraction when a ray of light hits the boundary between two materials. The Fresnel equations and finally the solutions of Maxwell's equations provided the connection between the incoming, the reflected and the refracted electromagnetic field, thus solving the problem of reflection and absorption in classical physics.

With the development of laser technology relativistic light intensities have become available, driving electrons nearly to the speed of light. Thus the problem of explaining reflection and absorption at these intensities is connected to the highly non-linear response of matter during its interaction with the laser radiation.

When a piece of solid is exposed to the radiation of a short ultra-intense laser pulse, the matter almost immediately turns into overdense plasma, consisting of an immobile ionic background and a hot electron fluid, which performs a complicated oscillatory motion. As a result of this non-linear interaction, a part of the incoming signal is reflected and can be observed in specular direction as a train of high harmonics, multiple of the incident laser frequency $\omega_0$.

The reflection of light from overdense plasmas at relativistic laser intensities (corresponding to dimensionless vector potential $a_0^2 \approx 1$) was explained by the Theory of relativistic spikes\(^{[1]}\), the most important result of which is that due to the non-linear electron dynamics inside the plasma, the spectrum of the reflected radiation is universal and obeys a power law.

In this article we consider the interaction of short laser pulses with overdense plasma in the relativistic regime. We will see that the reflection and generation of high harmonics in this regime result from the collective and purely relativistic effect of electron transport by the zeroes of the vector potential. Moreover, this gathering mechanism is also responsible for the relativistic absorption, thus providing a close connection between the two phenomena and explaining the generation of high energy electrons at the rear side of thin foils.

Reflection: coherent X-rays from relativistic plasmas
As it was first predicted analytically by Baeva et al.\(^{[1,2]}\) the signal reflected from plasma during laser-overdense plasma interaction in the relativistic regime contains a large number of coherent frequencies (high harmonics of the initial laser frequency), the spectrum of which depends neither on the details of the laser-plasma interaction nor on the plasma surface roughness. The harmonic intensity $I(\omega)$ decays according to a power law ($I(\omega)\sim \omega^{-8/3}$) up to frequencies of the order of $a_0^3$, where it starts rolling down exponentially. Figure 1 visualizes these theoretical results. The universal harmonic spectrum was confirmed experimentally at RAL with the spectacular observation of over 3200 harmonics, reaching for the first time keV X-ray energies and sub-nanometer wavelengths, despite surface roughness\(^{[3]}\).

It should be emphasized that mirror models relying on the relativistic Doppler Effect cannot explain the nature of the observed radiation (for detailed explanation of the drawbacks of such models see\(^{[2]}\)). In order to understand the generation of such a high number of coherent X-rays, one should look at the microscopic processes inside the plasma instead and clarify which electrons are responsible for emitting the high frequency radiation.

We consider, for the sake of simplicity, a laser pulse of frequency $\omega_0$, normally incident onto a slab of overdense plasma with surface in the $(y,z)$-plane. An electron inside the plasma skin layer has transverse momentum

$$p_x = \frac{e A_x(t,x)}{c},$$

Figure 1. Spectra of high harmonics generated during laser-overdense plasma interaction. The PIC simulations demonstrate the universal power-law decay of the harmonic intensity\(^{[1]}\).
where $A_\tau$ is the component of the vector potential tangential to the plasma. The normal velocity of the electron is

$$v_x = \frac{cp_x}{\sqrt{p_x^2 + c^2 A / c}}$$

(for $p_x \gg m_e c$). We see that at the point where $A_\tau = 0$, the skin layer electron moves with velocity $c$ towards the laser pulse. It was shown in [2] that the zero point of the vector potential moves with velocity $c$ from plasma to vacuum inside the skin layer. As a result, this zero collects relativistic electrons with positive momentum on its way. These electrons are responsible for the radiation of high energy harmonics (Figure 2).

However, the velocity of the zeros of the vector potential equals the phase velocity of light (detailed non-linear treatment of the question can be found in [2]). For this reason, the phase velocity of the radiation source (the collected relativistic electrons around the zero of the vector potential) coincides with the phase velocity of the generated radiation, leading to accumulation of radiation in phase inside the whole plasma skin layer as $A_\tau$ propagates through it.

Since the whole relativistic skin layer is involved in the process, the harmonic generation is robust and is not affected by the surface roughness, as long as this roughness does not destroy the skin layer structure. This explains the experimental observation of Ångstroem harmonics [3], which would have been prohibited by the phase matching condition if the harmonics had been generated at the very surface of the plasma.

The radiation accumulated during the time the zero of the vector potential travels inside the plasma skin layer manifests itself in the form of electromagnetic “shock” waves [2] propagating in vacuum (Figure 3). The high harmonics are locked inside the shocks and their electric field depends on time as

$$E_x(t, x) = \text{const} + \text{const} \times (ct - x)^3$$

Note that the shocks appear twice per laser period, coinciding with the zeros of $A_\tau$.

If one filters out the low frequency harmonics in Figure 3 one sees that in the time domain the high harmonic signal appears in the form of short pulses. Note that the finite duration of these pulses is due to a finite mismatch between the velocity of the vector potential zero and the velocity of relativistic electrons collected by this zero and co-moving with it. As $a_0$ grows this mismatch decreases causing the generation of ultra-short X-ray pulses for relativistic intensities ($a_0^2 \gg 1$) of the incident laser radiation.

**Physical mechanism of X-ray generation**

One can best visualize the physical picture of high harmonic generation by considering the electron distribution function $f(t, x, p_x)$ as a function of time, $x$-coordinate in the direction of the laser pulse propagation and corresponding electron momentum $p_x$ (Figure 4). Following the $r$-axis one can observe how the unperturbed plasma changes with time. It can be clearly seen that at certain moments of time (twice per laser period) the zeros of the vector potential collect electrons and drive them out of the plasma with velocity $c$. Note that these zero points arise inside the plasma and propagate in the direction of the vacuum, gathering and pulling the relativistic electrons on their way (Figure 2).

![Figure 2. The vector potential zero moves with velocity $c$ out of the plasma slab. On its way it collects electrons with coinciding velocity and transports them into vacuum.](image)

![Figure 3. (a) Incoming laser radiation. (b) Numerical PIC simulation ($a_0=20$, electron density equal to 90 times the critical plasma density) demonstrates the reflected radiation containing electromagnetic shocks [2].](image)

![Figure 4. Electron distribution function $f(t, x, p_x)$ as a function of time, $x$-coordinate in direction of laser pulse propagation and corresponding electron momentum $p_x$ [4].](image)
way out. As a result of this motion driven by the vector potential zero the electrons radiate high energy harmonics.

It should be emphasized at this point that the X-ray generation is not a result of synchrotron radiation by the single electrons. Instead, the whole plasma skin layer is in a coherent state radiating high harmonics. This is the result of the purely relativistic effect of electron gathering by the zero points of the vector potential.

*Relativistic absorption: fast electron injection into plasma*

The same physical process of electron injection by the zero points of the vector potential is responsible for the generation of high energy electrons, which transport energy inside the plasma. Indeed, the electrons collected by the zero points of the vector potential and driven out of the plasma are turned back by the Coulomb attraction to the plasma ions. After changing the sign of electron velocity the Coulomb force accelerates these electrons causing a shower of high energy electrons bombarding the plasma surface and penetrating into the bulk of the plasma. The high energy electron shower can be seen in Figure 4 as a foggy veil directed at an angle of about 45° in the \((x, t)\) plane, corresponding to electron velocities reaching about the speed of light.

In order to illustrate the geometry of electron propagation let us consider oblique laser incidence (Figure 5), for which the relativistic regime is reached when \(a_0^2 \cos^2 \Theta \gg 1\). In this case, the vector potential zero propagates in the plasma bulk in the direction in which the reflected light is emitted from the plasma. Consequently this is the direction in which the electrons, which are collected by the zero point, are transported out of the plasma.

On their way back, the electrons are driven by the Coulomb attraction to the plasma ions, causing the electron transport in the direction of laser incidence. For thin plasma foils, this flux of electrons moving in the direction of laser incidence should be clearly seen in experiments.

It is worth emphasizing that a flux of electrons moving in oblique direction cannot be explained by a model of plasma driven by the laser light pressure. Since the plasma frequency of overdense plasma is much higher than the frequency of the incidence laser light, one could expect that the plasma follows the applied laser pressure adiabatically. Electron transport by the zeros of the vector potential shows why this is not the case and highlights the deficiency of the model of laser pressure driven plasma.

Thus, the picture introduced above gives an important insight into the process of the relativistic absorption. Firstly, it explains how hot electrons moving with energies proportional to \(a_0\) and injecting energy into the plasma bulk are generated. Secondly, this approach shows that the bunches of high energy electrons can be much shorter than the laser light period. Thirdly, since the vector potential passes through zero twice per laser period, the theory predicts the generation of second harmonic radiation from the rear of the plasma target. And finally, the mechanism of electron transport by the zero points of the vector potential predicts the direction at which the energy is transported by high energy electrons into the plasma (Figure 5).

**Conclusions**

In this paper we saw that the physical mechanism responsible for the generation of high harmonics from overdense plasma in the relativistic regime is also responsible for absorption and injection of high energy electrons into the plasma. The theory predicts that at the rear side of the plasma slab there is a flux of fast electrons, which are not a result of light pressure action on the plasma. Instead, these electrons are driven out of the front side of the plasma by the zero points of the vector potential and are later pulled back by the Coulomb attraction to the ions to leave the rear side of the plasma in the direction of laser pulse propagation.

**Acknowledgements**

I am grateful to Dr. S. Gordienko for valuable discussions.

**References**


---

Figure 5. Trajectory of the relativistic plasma electrons transported out of the plasma by the vector potential zero and pulled back by the Coulomb attraction to the plasma ions, for oblique laser incidence (relativistic regime is reached for \(a_0^2 \cos^2 \Theta \gg 1\)). Electron flux in direction of laser propagation can be observed at the rear side of thin foils.