Dense plasma heating by inverse Bremsstrahlung

A. Grinenko and D. O. Gericke
Centre for Fusion, Space and Astrophysics, Department of Physics, University of Warwick, Coventry CV4 7AL, UK

Introduction
It is crucial for the design and critical evaluation of targets for inertial confinement fusion to thoroughly understand the interaction of the laser radiation with dense, strongly coupled plasmas. To accommodate the symmetry conditions needed, the absorption of laser energy must be carefully determined starting from the early stages\textsuperscript{[1,2]}. The absorption data for dense plasmas are also required for fast ignition by ultra-intense lasers due to creation of plasmas by the nanosecond pre-pulse\textsuperscript{[3]}. Least understood are laser-plasma interactions that involve strongly coupled $\Gamma>1$ and partially degenerate electrons. Such conditions also occur in warm dense matter experiments\textsuperscript{[4,5]} and laser-cluster interactions\textsuperscript{[6,7]}.

The dominant absorption mechanism for lasers with the parameters typical for inertial confinement fusion is inverse bremsstrahlung. This problem was first investigated by Dawson & Oberman for weak fields\textsuperscript{[8]}. Their approach was later extended to arbitrary field strengths by Decker et al.\textsuperscript{[9]}. However, due to the use of the classical kinetic theory, their results were inapplicable for dense, strongly coupled plasmas. This problem was addressed using a rigorous quantum kinetic description applying the Green’s function formalism\textsuperscript{[10,11]} or the quantum Vlasov approach\textsuperscript{[12]}. However, these approaches are formulated in the high-frequency limit which requires the number of electron-ion collisions per laser cycle to be relatively small. In the weak field limit, a linear response theory can be applied and thus the strong binary collisions were also included into a quantum description\textsuperscript{[13,14]} in this limit.

For dense strongly coupled plasmas, the approach for the evaluation of the laser absorption in both the high and low frequency limits must be fundamentally different. In the high-frequency limit, the electron-ion interaction has a collective rather than a binary character and the laser energy is coupled into the plasmas via the induced polarization current. On the other hand, binary collisions dominate laser absorption in the low-frequency limit resulting in a Drude-like formulation. At the intermediate frequencies, both strong binary collisions and collective phenomena have to be considered simultaneously. Interestingly, such conditions occur for moderate heating at the critical density of common Nd:Yag lasers.

Here, we discuss a novel method to calculate collisional absorption that bridges between the high- and low-frequency limits and incorporates weak collective interactions as well as strong binary collisions. To this end, the interactions in the kinetic equation are formally split into weak and hard collisions. The former are equivalent to the average Hartree field and can be considered as a linear collective polarization response of the system to the external field. The latter can be shown to be equivalent to an average friction force between the electron and ion fluids and can be treated as the stopping power of ions in the electron fluid. This statement is true for laser frequencies in the important range around the plasma frequency, where the electrons and ions have enough time to interact as two fluids. Thus, the link between the two basic energy absorption mechanism, the stopping power and collisional absorption, is established. This allows one to apply well developed models for the stopping power (see, e.g., Refs.\textsuperscript{[15-17]} to the problem of laser absorption.

One crucial improvement of our method that allows avoiding the restrictions with respect to the laser frequency is the transformation of the problem to the generalized rest frame in which the electrons distribution can be linearized. However, the latter is not assumed to be the frame of the freely oscillating electrons as in previous works\textsuperscript{[8-11]}, but is determined by the driving field and the friction between the electron and ion species. Therefore, the description of the collective electron response using the expansion in the electron rest frame is almost unchanged from earlier approaches\textsuperscript{[9,12]}, but the use of the generalized rest frame modifies the final result. The use of the quantum mechanical formalism allows avoiding the use of ad hoc cutoffs and the method stays reliable for strong electron-ion interactions and degenerate electrons. The agreement of our results with molecular dynamics (MD) simulations\textsuperscript{[18-20]} in the parameter range examined validates the few assumptions made in the derivations.
Outline of the theory

Collisional absorption of laser energy is commonly characterized in terms of a frequency-dependent electron-ion collision frequency defined as \[ \nu_{ei}(\omega) = \frac{4\pi\omega^2}{\omega_p^2} \left( \mathbf{j} \cdot \mathbf{E} \right) \triangleq \frac{4\pi\omega^2}{\omega_p^2} \text{Re}(\sigma(\omega)) \] (1)

where the brackets denote the average over one period of the laser field. The collision frequency is therefore determined by the electron current which is, in turn, given by the first moment of the electron distribution function. Therefore, quantum kinetic equation for the electron distribution has to be solved using the methods described in [11]. The resulting solution is

\[ \nu_{ei}(\omega) = \frac{\gamma_{ei}^2}{\omega_p^2} - \sum_{n=0}^{\infty} \sum_{n'=0}^{\infty} \int_0^\infty dk^2 D(k)(n+m)\omega(\omega - \nu_{ei}) S_i(k) \] (2)

Here, \( \nu_{ei} \) is the frequency of the hard collisions, \( \omega_0 \) is the laser frequency, \( \omega_p \) is the plasma frequency, \( S_p \) is the ion-ion structure factor, \( \gamma = 1 + \frac{\nu_{ei}}{\omega_p^2} \), \( D(k,\omega) \) is the RPA dielectric function and \( J_\nu \) are the Bessel functions of the first kind. The parameter \( \alpha = (\omega_0\omega_p/2\pi^2)Z^2e^2/mv^2 \gamma \) is proportional to the charge number \( Z \) and inverse proportional to the square of the free quiver velocity. In Eq. (2), the first term describes the absorption due to the strong collisions that corresponds to the Drude conductivity. The second term describes the collective dynamic absorption term due to the polarisation current.

Outline of the theory

Collisional absorption of laser energy is commonly characterized in terms of a frequency-dependent electron-ion collision frequency defined as [20]

\[ \nu_{ei}(\omega) = \frac{4\pi\omega^2}{\omega_p^2} \left( \mathbf{j} \cdot \mathbf{E} \right) \triangleq \frac{4\pi\omega^2}{\omega_p^2} \text{Re}(\sigma(\omega)) \] (1)

where the brackets denote the average over one period of the laser field. The collision frequency is therefore determined by the electron current which is, in turn, given by the first moment of the electron distribution function. Therefore, quantum kinetic equation for the electron distribution has to be solved using the methods described in [11]. The resulting solution is

\[ \nu_{ei}(\omega) = \frac{\gamma_{ei}^2}{\omega_p^2} - \sum_{n=0}^{\infty} \sum_{n'=0}^{\infty} \int_0^\infty dk^2 D(k)(n+m)\omega(\omega - \nu_{ei}) S_i(k) \] (2)

Here, \( \nu_{ei} \) is the frequency of the hard collisions, \( \omega_0 \) is the laser frequency, \( \omega_p \) is the plasma frequency, \( S_p \) is the ion-ion structure factor, \( \gamma = 1 + \frac{\nu_{ei}}{\omega_p^2} \), \( D(k,\omega) \) is the RPA dielectric function and \( J_\nu \) are the Bessel functions of the first kind. The parameter \( \alpha = (\omega_0\omega_p/2\pi^2)Z^2e^2/mv^2 \gamma \) is proportional to the charge number \( Z \) and inverse proportional to the square of the free quiver velocity. In Eq. (2), the first term describes the absorption due to the strong collisions that corresponds to the Drude conductivity. The second term describes the collective dynamic absorption term due to the polarisation current.

Here, we will focus on the calculation of the hard collision term. It is calculated using the stopping power as

\[ -\frac{e}{m} \int dp \ p \ j_{ei}^i = \frac{e}{m} \frac{\delta(E_i)}{\delta x}(V) = \frac{e}{m} R(V) = \nu_{ei} j \] (3)

The hard-collision frequency given by the latter formula can be represented using the Gould-DeWitt as a sum of three contributions [19]

\[ \nu_{hc} = \nu_T + \nu_{LB} - \nu_B \] (4)

where the first term corresponds to the static T-matrix representing the strong coupling, the second term is the dynamic Lenard-Balescu contribution and the last term is the static Born contribution that is subtracted to prevent double counting. However, since the dynamic effects in the total collision frequency are already included in the polarization current contribution, this form is inapplicable here, because it contains the weak dynamic effects in the Lenard-Balescu term whereas we are interested in hard-collisions only. However, the last form is especially useful since for small relative species velocities \( V < v_{th} \), the stopping power is a linear function of the velocity and it can be determined using the simple fit formula

\[ \nu_{hc}[\text{th.u.}] = \exp(-0.0735x^2 + 1.337x - 1.8511) \]

where the collision frequency is given in thermal units [19] and \( x = \frac{Z^2e^2}{\sqrt{m}v^2} \) is the dimensionless similarity parameter.

The parameter \( \alpha = (\omega_0\omega_p/2\pi^2)Z^2e^2/mv^2 \gamma \) is proportional to the charge number \( Z \) and inverse proportional to the square of the free quiver velocity. In Eq. (2), the first term describes the absorption due to the strong collisions that corresponds to the Drude conductivity. The second term describes the collective dynamic absorption term due to the polarisation current.

Here, we will focus on the calculation of the hard collision term. It is calculated using the stopping power as

\[ -\frac{e}{m} \int dp \ p \ j_{ei}^i = \frac{e}{m} \frac{\delta(E_i)}{\delta x}(V) = \frac{e}{m} R(V) = \nu_{ei} j \] (3)

The hard-collision frequency given by the latter formula can be represented using the Gould-DeWitt as a sum of three contributions [19]

\[ \nu_{hc} = \nu_T + \nu_{LB} - \nu_B \] (4)

where the first term corresponds to the static T-matrix representing the strong coupling, the second term is the dynamic Lenard-Balescu contribution and the last term is the static Born contribution that is subtracted to prevent double counting. However, since the dynamic effects in the total collision frequency are already included in the polarization current contribution, this form is inapplicable here, because it contains the weak dynamic effects in the Lenard-Balescu term whereas we are interested in hard-collisions only. However, the last form is especially useful since for small relative species velocities \( V < v_{th} \), the stopping power is a linear function of the velocity and it can be determined using the simple fit formula

\[ \nu_{hc}[\text{th.u.}] = \exp(-0.0735x^2 + 1.337x - 1.8511) \]

where the collision frequency is given in thermal units [19] and \( x = \frac{Z^2e^2}{\sqrt{m}v^2} \) is the dimensionless similarity parameter.

The parameter \( \alpha = (\omega_0\omega_p/2\pi^2)Z^2e^2/mv^2 \gamma \) is proportional to the charge number \( Z \) and inverse proportional to the square of the free quiver velocity. In Eq. (2), the first term describes the absorption due to the strong collisions that corresponds to the Drude conductivity. The second term describes the collective dynamic absorption term due to the polarisation current.

Here, we will focus on the calculation of the hard collision term. It is calculated using the stopping power as

\[ -\frac{e}{m} \int dp \ p \ j_{ei}^i = \frac{e}{m} \frac{\delta(E_i)}{\delta x}(V) = \frac{e}{m} R(V) = \nu_{ei} j \] (3)

The hard-collision frequency given by the latter formula can be represented using the Gould-DeWitt as a sum of three contributions [19]

\[ \nu_{hc} = \nu_T + \nu_{LB} - \nu_B \] (4)

where the first term corresponds to the static T-matrix representing the strong coupling, the second term is the dynamic Lenard-Balescu contribution and the last term is the static Born contribution that is subtracted to prevent double counting. However, since the dynamic effects in the total collision frequency are already included in the polarization current contribution, this form is inapplicable here, because it contains the weak dynamic effects in the Lenard-Balescu term whereas we are interested in hard-collisions only. However, the last form is especially useful since for small relative species velocities \( V < v_{th} \), the stopping power is a linear function of the velocity and it can be determined using the simple fit formula

\[ \nu_{hc}[\text{th.u.}] = \exp(-0.0735x^2 + 1.337x - 1.8511) \]

where the collision frequency is given in thermal units [19] and \( x = \frac{Z^2e^2}{\sqrt{m}v^2} \) is the dimensionless similarity parameter.
References
23. A. Grinenko, D. O. Gericke, to be published.