3D ray tracing of high intensity laser beam through plasma guiding structures

Contact *scott.martin@univ.ox.ac.uk*

S. A. Martin

University College,
Oxford,
OX1 4BH
United Kingdom

D. Neely

Central Laser Facility,
STFC Rutherford Appleton Laboratory,
OX11 0QX,
United Kingdom

C. D. Armstrong[1,2]

[1] Department of Plasma Physics,
University of Strathclyde, Glasgow,
G4 0NG,
United Kingdom

[2] Central Laser Facility,
STFC Rutherford Appleton Laboratory,
OX11 0QX,
United Kingdom

Abstract

High power lasers are challenging to focus down to useful intensities using normal optical components. This project seeks to use the optical properties of plasma to do this. A 3D ray tracing program is developed using MATLAB in order to carry out a study of how a high power, short pulse laser beam may travel through a plasma guiding structure. The motivation for this is the prospect of confining the beam sufficiently to increase the intensity potentially by a factor of up to 10. Wave optics effects are not treated here but it is hoped this geometrical study will provide some insight into laser confinement by short-lived plasmas.

Introduction

This project seeks to use geometric optics to provide an approximate optimization of the laser guiding. At the Central Laser Facility (CLF), operated by the Science and Technology Facilities Council (STFC), the Vulcan laser produces a petawatt laser pulse for a pulse duration on the order of 1 picosecond. Lasers such as these are used for a wide range of physics experiments with applications and so it is vital that such high-power beams can be controlled with a great deal of precision. CLF use lasers to ionize and accelerate electrons from a target, thus creating a plasma. High power pulses are preceded by amplified spontaneous emission (ASE), light produced by spontaneous emission that is then amplified by the gain medium. This ASE can then be used to expand the target and change the interaction parameters of the plasma. Plasma has the interesting property that its refractive index is less than one, meaning that a light ray that enters a region of plasma will bend back away from the plasma (treating the ray's propagation geometrically using Snell's law). This causes a plasma surface to act as a mirror. The refractive index of a plasma can be related to the electron density (N) and the critical electron density at which light stops propagating (N_c), due to the plasma frequency becoming greater than the frequency of the incident light:

$$N_c = \frac{4\pi^2 m_e}{\mu_0 e^2 \lambda^2} \tag{1}$$

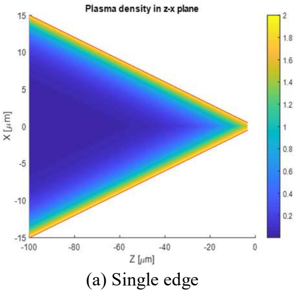
$$n(\boldsymbol{r}) = \sqrt{1 - \frac{N(\boldsymbol{r})}{N_c}} \tag{2}$$

The aim of this project is to consider a 3D plasma guiding structure (such as a hollow cone) which the beam could be sent through to focus it down to a smaller and more intense spot than could be achieved with conventional optical components. For the purposes of this investigation we shall treat the light purely geometrically and ignore temporal effects (such as distortion of the plasma by the front of the laser pulse) since this is just a preliminary study to get a working understanding of how such systems behave and whether it could be feasible to focus the light using these guiding structures. The assumption about temporal effects was judged to be reasonable at this stage since the pulse duration is typically on the order of 1 ps and the guiding structures are short enough that the time of flight of each light ray is negligible enough that optical path difference down the structure can be ignored.

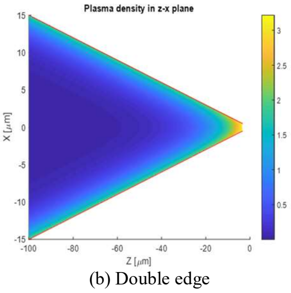
The electron density of a region of induced plasma is generally taken to decay exponentially away from the surface at which its density is at its maximum, N_s (i.e. at the surface of the material that was ionized by the ASE). The rate at which the electron density drops off with the distance (r) away from the maximum density surface is characterized by the scale length (L_s) of the plasma:

$$N = N_s e^{-\frac{r}{L_s}} \tag{3}$$

This project considered two different plasma density models; a single edge and a double edge model. For the single edge model, the density of the plasma was taken to be exponentially decaying with the distance to the closest point on the surface of the original guiding structure (e.g. the cone), whereas for the double edge model, the density was taken to be the addition of the decaying density from the closest wall added to the decaying density from the opposite wall. The double edge model is a more physically realistic model since the guiding structures are on a small enough scale that the scale length of the plasma is large enough relative to the structure that the effect of the plasma from the opposite wall cannot be neglected.



(a) Single edge



(b) Double edge

Figure 1: Density models applied to a straight-sided cone (where density is measured in units of N_c and N_s=2N_c)

Developing the ray tracing program

Tracing individual rays

The 3D ray tracing was developed based on an iterative calculation using Snell's law to calculate the new trajectory a ray should take at each step. In order to make this method work in

3D for any general plasma density profile it was important to find a method for calculating the local normal relative to which the angles involved in Snell's law should be measured. This normal should point in the direction of greatest increase in refractive index, and so can be calculated using ∇n. For the program to be versatile enough to deal with any given plasma profile it was necessary to estimate the value of ∇n at a given location by constructing a grid of 6 points around the location separated by a small distance in each coordinate direction respectively. The refractive index was then calculated at each of these six points and the difference in n caused by travelling a small direction in each coordinate direction was then used to construct an estimate for ∇n.

Another issue that required consideration during the development of the program was the fact that the simple refraction model breaks down at the point where the ray turns and starts to leave the plasma, since this model would predict that the light would tend asymptotically to travel parallel to the plasma surface which is not what is observed physically. To get around this a small angle dθ was defined such that if the ray was measured to be this small angle away from 90° to the normal, the ray will be reflected out at this same angle.

Finally, the step size had to change dynamically as the simulation progressed since a very small step size was required close to the turning point but was computationally prohibitive if used throughout. Two methods were used to achieve this changing step size. Firstly, a scaling formula that Myron Huzan *et al.* [1] devised to scale the step size exponentially depending on the plasma density was utilized.

$$dr = dr_{max}\eta^{1-\frac{N_c}{N_c-N(r)}} \qquad (4)$$

In addition, a condition that checked to see if the ray had overshot the point at which it would have turned around if solved analytically (at which point Snell's law breaks down and predicts an angle of refraction that is complex) and reduced the previous step size repeatedly until the point doesn't lie beyond the turning point was used.

Test Case

To test that the simulation was working, it was applied to the simple case of monochromatic light incident on a plasma density that decays exponentially with z (i.e. a planar mirror on the x-y plane). In this setup you would expect a ray to approach and recede from the mirror at the same angle, since it should behave similarly to a conventional mirror. You can also derive a formula for the electron density at the turning point N_e as a function of the angle of incidence:

$$N_e = N_c \cos^2\theta_i \qquad (5)$$

Using these two expectation values, the results of the simulation were sampled for these two quantities at a range of angles of incidence. The errors in these quantities were found to be very small (fractional errors of between 10^{-10} and 10^{-7}. At near normal incidence there are less iterative steps and as a result the errors are larger, however, at larger angles the code performs remarkably well with errors of order 10^{-9}.

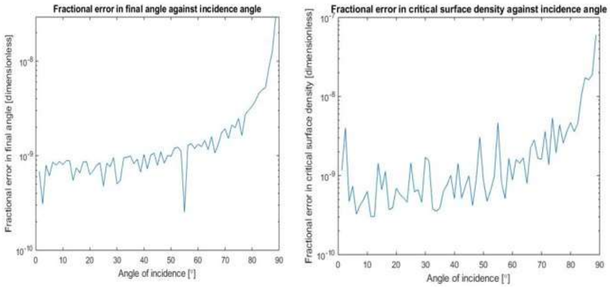


Figure 2: Fractional errors in angle and plasma density at turning point for simple planar mirror (note that the errors get larger as you approach 90° but remain very small.

Constructing a beam of rays

In order to simulate the beam as it enters the plasma cone, it is necessary to use multiple individual light rays together to simulate the profile of a beam. Without this consideration the changing width of the beam could not be studied. If the beam is assumed to enter the guiding structure at its most focused point, this focal spot will have a finite radius, R_s, defined by the beam's wavelength and its f-number (ratio of the focusing system's focal length to its aperture diameter, denoted by $f_\#$) using the Rayleigh limit for circular apertures:

$$R_s = \frac{1.22\lambda}{f_\#} \qquad (6)$$

In the absence of any guiding structure the beam will then diverge from the focal spot with a radius defined by:

$$w(z) = R_s\sqrt{1+\left(\frac{z}{z_R}\right)^2} \qquad (7)$$

$$z_R = \frac{\pi R_s^2}{\lambda}$$

Z_R is the Rayleigh length which is defined as the distance in the direction of propagation from the beam waist to the point where the cross-sectional area has doubled. In order to approximate this curved beam waist profile with straight rays a formula for the maximum angle at which a ray can be sent such that it stays within the beam waist as a function of distance (d) from the center of the spot can be found. This formula was arrived at by forcing the tangent to the waist curve to pass through the desired point on the focal spot.

$$\phi_{max}(d) = \arctan\left(\frac{d}{z_R}\sqrt{\frac{R_s^2}{d^2}-1}\right) \qquad (8)$$

Given this result, a beam of rays can then be approximated in 3D by using a distribution of ray origins across the focal spot and then sending rays at a range of angles constrained by ϕ_{max}.

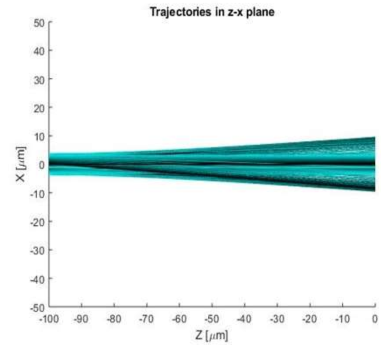


Figure 3: 2D slice of beam diverging from focal point (no guiding structure present). The ray colours scale from black to blue with the distance from the center of the focal spot the ray was emitted from

Modelling energy loss and intensity

To study the intensity of the beam, a measure of the energy of the simulated beam is required. In this study this is modelled simply by applying a weighting to each ray and defining the energy at a given z coordinate (where z is the direction of travel down the guiding structure) as being the sum of the weights of each ray passing that coordinate.

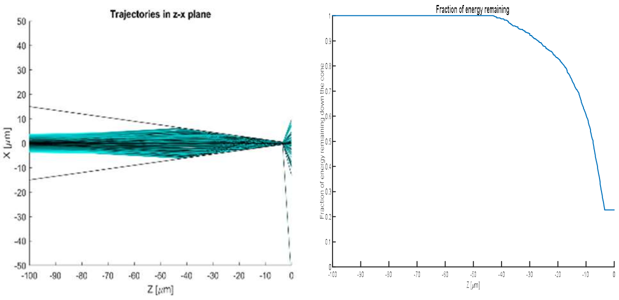
It is also important to introduce an energy loss mechanism into the simulation since the conservation of energy dictates that as the plasma confines the light beam some of the beam's energy will be lost to the plasma. Laser absorption is a multi-faceted process that is typically explored in vast particle-in-cell simulations. In order to be able to compute this within the ray-tracing code we approximated the energy loss by a fixed percentage at each point of inflection. The percentage reduction applied was increased with the scale length of the plasma to make this more physically realistic. This model has drawbacks but was judged to provide a sufficiently accurate characterization of the effect of the underlying physical processes. From this energy definition intensity was then simply taken to be the energy at a given z coordinate divided by the cross-sectional area of the confined beam.

In all the following simulations the trace of a given ray was stopped if it started to return out towards the cone entrance, allowing us to consider only the energy of the forward propagating beam.

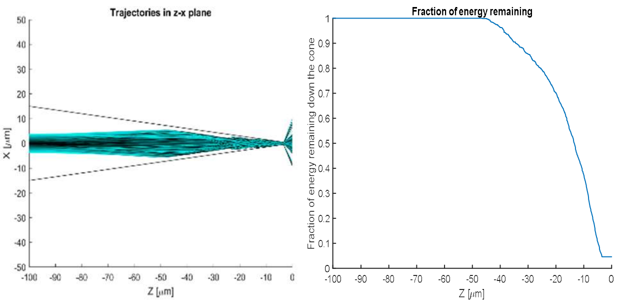
After much refinement, the MATLAB code was sufficiently efficient to test a wide range of plasma and laser parameters allowing for optimum geometries to be configured. Outlined in the following sections are some specific test cases and findings.

Results

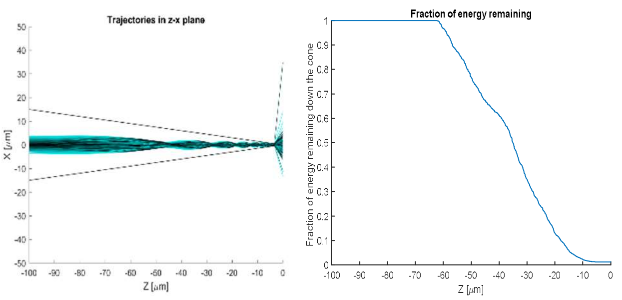
Throughout all the simulations in this project λ was taken to be 1.053μm, since this is the operational wavelength of CLF's Vulcan laser. This can then be used in Eq. 1 to define the critical density.



(a) L_s=0.1μm



(b) L_s=0.5μm



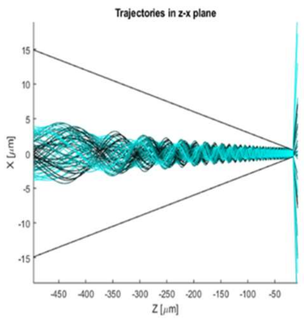
(c) L_s=2μm

Figure 4: Ray trajectories and corresponding energy decay of f_3 beam entering a cone with an opening aperture diameter of 15μm and length of 100μm with varying plasma scale lengths using the single edge plasma density model

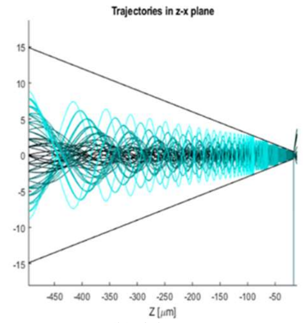
Straight-sided cone using single edge plasma density model

Firstly, the effect of varying the scale length of the plasma was considered. The graphs in Fig. 4 show the profile of an f_3 beam entering a hollow straight-sided cone where the plasma was modelled using the single exponential model with the surface density set to be the critical plasma density. You can see clearly that as the plasma scale length is increased a greater confinement of the beam is achieved. However, Fig. 4 also shows the energy of the beam reducing more for the larger scale lengths counteracting the greater confinement effect, which will mitigate the potential increase in intensity.

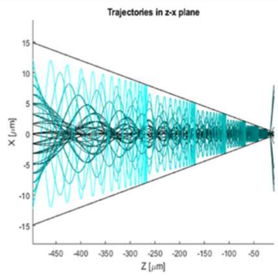
Next, the effect of changing the f-number of the incident beam was considered. A higher f-number beam will have a larger spot of maximum focus, but the light will diverge at a smaller angle. In Fig. 5 you can see that for the higher f-numbers the focal spot size is large enough that the outer rays of the beam enter the cone close to the cone sides and so are immediately diverted sharply towards the cone axis. For these highly confined outer rays we see that they stop (reflect enough that they start to return out of the cone) progressively earlier down the cone with the distance of the ray origin to the center of the focal spot. Again, there is a price for this higher confinement for larger f-numbers as the energy of these beams drop more quickly down the cone. With these energy considerations in mind the f_3 beam was used when investigating other parameters.



(a) f_3 beam



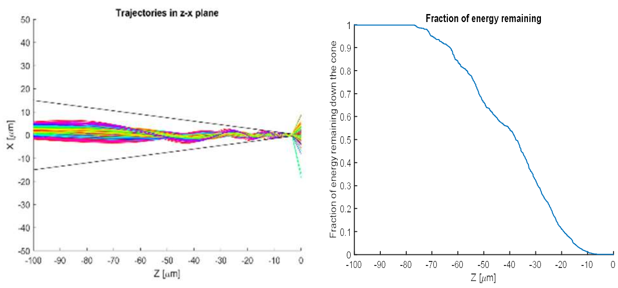
(b) f_7 beam



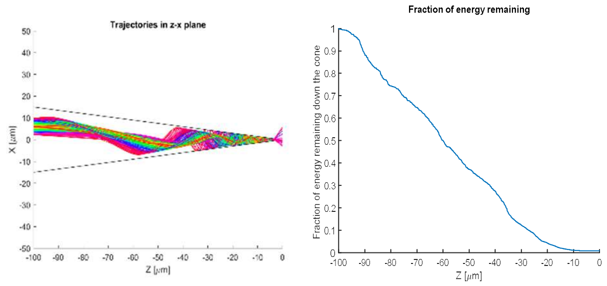
(c) f_{10} beam

Figure 5: Ray trajectories for various f-numbers in one plane

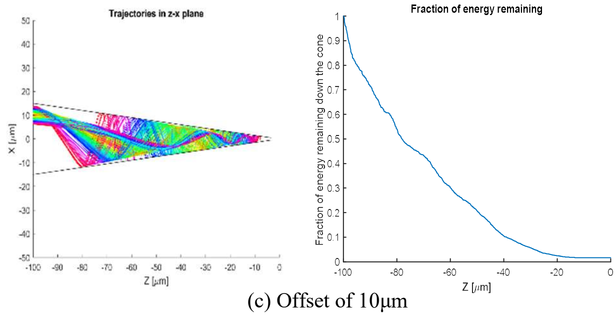
Finally, the effect of the beam being sent in at an offset from the cone axis was considered. This situation is of great practical importance since in any real-world experimental setup, it is highly likely that this offset cannot be assumed to be zero, even given the precision equipment and techniques of CLF. The trajectories for these offset beams are shown in Fig. 6 and show how the beam propagates less far down the cone as it is sent it at increasing offsets, however, for small offsets it seems that the beam still exhibits fairly strong confinement behaviour. As you might expect the energy drops away more rapidly as you increase the offset (also shown in Fig. 6).



(a) Offset of 2μm



(b) Offset of 6µm



(c) Offset of 10µm

Figure 6: Ray trajectories and corresponding energy decays for f_3 beam inputted into cone at an increasing distance from the cone axis

Straight-sided cone using double edge plasma density model

Given this better understanding of beam confinement's dependence on the various parameters of the configuration a more realistic picture can now be attained using the double exponential model described earlier for the plasma profile. The surface density of the plasma is also here increased (to roughly 10 times the critical density) to make the configuration more applicable to experiments. A particularly high confinement case is shown in Fig. 7. It is important to keep in mind that the whole beam stopping before the end of the cone used is not necessarily a problem since a truncated cone could easily be produced such that the region of maximum light intensity coincides with the guiding structure's output.

For this more realistic, high confinement case we can now consider how the intensity varies down the cone to determine the enhancement in intensity. Fig. 7 also shows the energy and intensity evolution of the beam down the cone. As the beam is transported down the cone, the initial confinement focusses the beam to below the original focal spot size without losing energy, increasing the effective intensity. As transport continues there are subsequent interactions that rapidly reduce both the size and energy of the spot. The physical meaning of the dramatic peak as the beam stop propagating entirely should be treated with scepticism, but the initial confinement and resulting intensity improvement is the region of interest.

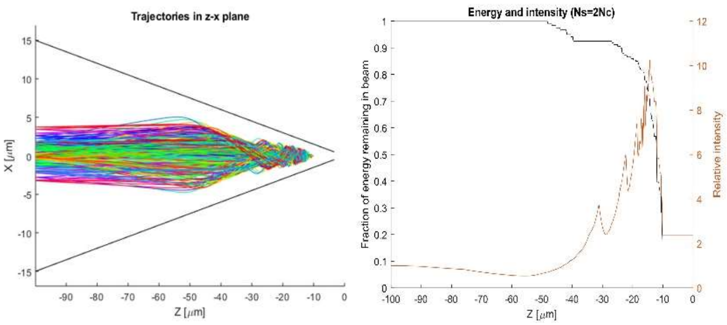


Figure 7: Trajectories and intensity for f_3 beam entering double edge plasma modelled cone with scale length 0.5µm. Note the light stops at boundary with over dense region

Conclusions

In conclusion, this study seems to suggest that conical plasma guiding structures can be used to confine a high power, short pulse laser to a sufficient degree to achieve a higher intensity than would otherwise be possible. However, additional physics could in future be included in order to improve the accuracy of this model.

A fully 3D code was developed with an absorption model included to test complex plasma parameters and the results for a wide variety of guiding structures. Initial tests calculating the achievable intensity were promising. Indicating a stable factor of 2 increase in intensity with a possible factor of 10 as the beam approaches the end of the structure. The code used was found to have very small errors when applied to the simple planar mirror configuration, giving a high degree of confidence in its findings for more complex configurations. The other positive aspect of this code is that is general enough in its functionality to be applied to more complex guiding structure shapes. For example, it was briefly applied to a trumpet-shaped guiding structure in the final days of the project.

Future studies should consider the physics of the laser's absorption more rigorously. Temporal effects should also be considered, since the front of the laser pulse may deform the plasma and affect the passage of the rear of the pulse.

Perhaps the largest omission in this study is to ignore all wave optics effects for the confined beam. This purely geometrical ray tracing approach cannot be expected to give the full picture; however, it provides a demonstration of concept and serves to give an understanding of how such an optical system would behave. The results of this study seem encouraging enough to imply this could be a fruitful field of study for CLF over the years to come.

Acknowledgements

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References

1. M Huzan et al. *Plasma Scale Length Effects on an Imaging Geometry*
2. C. Thaury et al. *Plasma Mirrors for Ultrahigh-intensity Optics* Nature Physics, 3.6 (2007): 424
3. D. Neely et al. *Enhanced proton beams from ultrathin targets driven by high contrast laser pulses* Applied Physics Letters 89.2 (2006): 021502