Time-dependant Nyquist limited Fourier transform analysis of electromagnetic pulse data

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During a high power laser-target interaction, electromagnetic pulses (EMP) are generated with frequencies on the order of magnitude of 10^3 GHz. Current oscilloscope capabilities are on the order of 10 GHz; far below the pulses being measured. Therefore oscilloscope diagnostics on these experiments are working at their bandwidth limits, and sampling lengths begin to limit the valid frequency output from existing binned (time-dependant) Fourier transform techniques. We describe a technique created around Nyquist's theorem on digital sampling rates of analogue signals, varying bin sizes of different frequency blocks to allow for time-dependant valid and broad spectra analysis from EMP measurements.

1 Introduction

A significant electromagnetic pulse (EMP) can be created during high power laser interactions of the Vulcan laser system [1] within the petawatt target chamber using solid targets (figure 1). EMP has long been a considerable limiting factor on electrical diagnostics of the laser-target interactions, in some cases causing a total loss of data and in others masking results within high noise profiles. In 2004, Mead *et. al.* attempted to characterise the EMP radiation, finding two principal frequencies of 63 MHz and 59 MHz in order of signal strength [2]. It was theorised that these arose from the modes of the metal cladded chamber, of dimensions 4.6 m north-south by 2.2 m east-west by 2.0 m vertically.

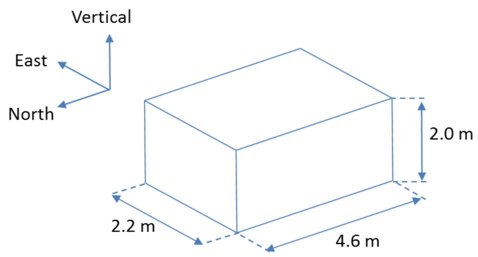

Figure 1: Diagram of the petawatt target chamber and its internal cavity dimensions without the metal clad shielding (25 mm). The laser-target interaction point is roughly situated to be the centre of the chamber.

Recent studies have been undertaken to further characterise the EMP radiation from target interactions in the petawatt target chamber using faster oscilloscopes that were previously unavailable. Preliminary results from this study shows low frequency modes in the voltage time oscilloscope traces (figure 2) growing after the initial burst of high frequency noise from the interaction itself. The emergence of these low frequency modes with a time delay was the justification for developing a time-dependant Fourier transform technique. The oscilloscope capabilities spanning over multiple GHz demand a Nyquist limited technique to be included, outlining the minimum and optimal sampling rates for the given frequency.

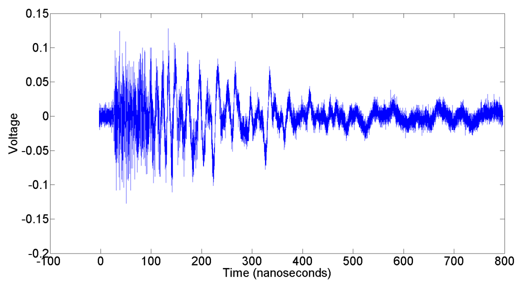

Figure 2: An example trace of EMP pick-up with a moebius loop [3] attached to an oscilloscope, displaying voltage as a function of time.

2 Theory

A standard Fourier transform converts data (f) from time-space (t) to frequency-space (k). The transformed data is denoted as $\tilde{f}$ instead of f for ease of understanding.

$$\tilde{f}(k) = \int_{-\infty}^{\infty} f(t) e^{2\pi itk} \mathrm{d}t \tag{1}$$

This is particularly useful for signal processing, as frequency information can give insight into the origin of the waveforms. However, computing systems by nature are unable to perform continuous calculus. Instead, a discrete form of the Fourier transform is used, where $j_{(t)}$ is one of n maximum points in time-space.

$$\tilde{f}(k) = \sum_{j_{(t)}=1}^{n_{(t)}} f(t) e^{\left(\frac{-2\pi i}{n_{(t)}}\right)(j_{(t)}-1)(k-1)} \tag{2}$$

In order to gain time parameters in frequency-space the data can be split into separate bins, and the Fourier transform of each computed. This will give a time resolution equal to the size of one bin in time, and a frequency resolution equal to twice the inverse of the time resolution. The factor of two arises from the Nyquist theorem encompassing digitising analogue signals. The theorem states that for any given signal, of maximum frequency f_{max}, the sampling frequency must be at least two times greater ($\geq 2f_{max}$) in order to prevent loss of bandwidth [4]. For this reason uniform bin analysis is not suitable over a large frequency range. The Nyquist theorem also predicts a deficit in a Fourier transforms amplitude output by a factor of $\frac{1}{2}$ that must always be corrected in analysis.

The optimal sampling length is two periods of oscillation, and with large frequency diversity of 10 MHz to 1 GHz the highest frequency components can undergo 10^4 periods in the same time-scale as 1 period for the lowest frequency components (10^{-5}s and 10^{-9}s respectively). Therefore if the bins are the length of 2 periods for low frequency oscillations the higher frequencies oscillate multiple times in a single bin. This outputs an average amplitude rather the change of amplitude over the time range, negating the use of a time-dependant technique being applied (figure 3). Conversely, shorter bins, used to obtain the time resolution necessary to analyse the high frequency waves, will have impaired frequency resolution that is unable to resolve the modal structures in the lower frequencies as they will not have completed the required number of oscillations in the time range of the bin (figure 4).

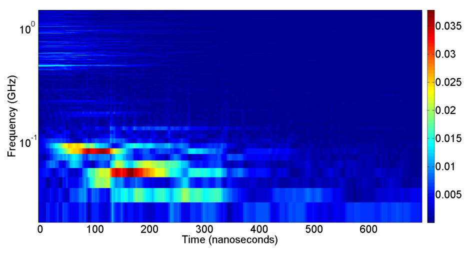

Figure 3: Time-dependant Fourier transform analysis of the EMP trace of figure 2, at bin length of 100 ns; optimal for low frequencies on the order of 10 MHz. The bin length is too long to adequately represent the high frequencies, and therefore any useful time information within this band is nullified.

Therefore a single pass uniform bin analysis is insufficient for the breadth of data acquired. Instead, a multi-pass system can be used, with the bin length varying with each pass and different frequency ranges extracted. As highlighted previously, obtaining the correct bin length is crucial for the effectiveness of this technique. A relation must be obtained to compute the optimal bin length from the frequency range tested and retain normality across each pass; the summation of the mean frequency of the frequency range and the bin length was kept constant throughout.

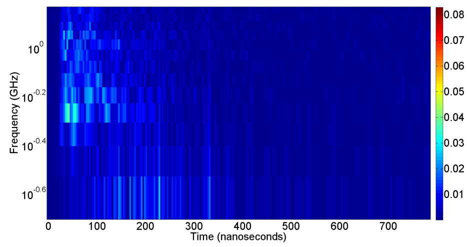

Figure 4: Time-dependant Fourier transform analysis of the EMP trace of figure 2, at bin length of 10 ns. The difference in y axis scaling with respect to figure 3 is due to the smaller bin size being unable to resolve frequencies on the same order of magnitude. This also accounts for the lack of the low frequency bursts seen in figure 3, as the EMP of lower frequencies has not undergone the necessary 2 period oscillation dictated by the Nyquist theorem.

3 Method

For a time-dependant Nyquist limited Fourier transform system, each data point is stored in a single cell inside of an array. The frequency range is determined by the experimental set up, rather than a mathematical construct.

The first bin of data (t_0) begins at $t = 0$ ns, with its size (l_0) is pre-defined by the desired initial frequency resolution ($\delta\nu_0$) and the sampling rate of the oscilloscope:

$$\frac{2}{\delta\nu_0 |t_1 - t_0|} = l_0 \tag{3}$$

A Fourier transform is performed on this bin and the normalised modulus of the resulting relative amplitude is saved to an array, along with the corresponding frequency of each cell. The start cell of the extracted data is then increased by a constant pre-defined value far below that of the bin size; this gives an effective moving bin window to the results. Analysis is then executed in an identical vein, and the results of the Fourier transform and its subsequent frequencies added onto the output array. This process is repeated until the next bin is out of bounds of the dataset.

At this point there is a 2 dimensional array of analysed data; the x index is representative of time, the y index is representative of frequency and each value is the relative amplitude in frequency space. A subsection of the 2 dimensional array is removed, where the frequencies are valid for the bin length used. Each cell is then divided through by the size of the bin used to analyse it, by way of normalisation across different bin sizes.

The process is repeated using bin lengths valid for different frequency ranges, and their results added to the valid frequency ranges already extracted. To standardise the bin sizes used, the product of the lowest frequency in the valid range ($\bar{\nu_n}$) and the bin size squared is always equal to that used for the first bin:

$$\bar{\nu_0} {l_0}^2 = \bar{\nu_n} {l_n}^2 \tag{4}$$

For ease of plotting, smaller arrays are padded to be the same size as the largest array. The padded value is 5% of the magnitude of the largest data point in the array, and negative. This allows for distinction of padding from output data whilst not adversely affecting the scales involved. All the arrays are concatenated together and plotted graphically with a 2 dimensional colour-map, displaying the modulus of the relative amplitude as a function of both frequency and time. As the motivation for this analysis was principally to further the work of Mead *et. al.* [2], the lower frequency results have been prioritised through the logarithmic y axis (figure 8).

The reference point for each bin analysis is the first cell, and its corresponding time value, rather than a median time across the bin.

4 Results and discussion

The technique was first applied to a simulated sinusoidal waveform (figure 5) by means of standardisation.

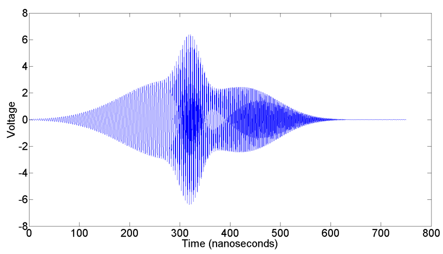

Figure 5: Test data created using a combination of sinusoidal waves and Gaussian peaks to produce a pre-determined waveform in intensity, frequency and time.

The time-dependant Nyquist limited Fourier transform analysis of figure 7 can be compared to the theoretical distribution of figure 6. It is evident from negligible differences between the two colourbars that the technique can accurately obtain the magnitude of the EMP frequencies, with the colour plots consistent over all three frequencies. The technique's frequency resolution is reduces in comparison to the theoretical; furthermore, there are traces of lower intensity artefacts protruding in both the positive and negative frequency directions from each frequency line detected. There is a constant time difference of the order of 10 nanoseconds, apparent in the total length of the analysis as well as the time occurrence of each frequency burst. This is likely to be a result of the bin length analysis of the technique,

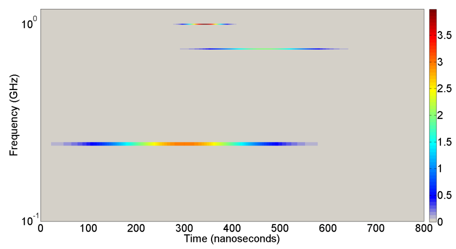

Figure 6: A theoretical frequency distribution as a function of time, created from the three sinusoidal wave and Gaussian peak combinations used to create figure 5. The colourscale has been altered to have a zeroth element coloured grey in order to highlight the total time durations of each frequency. The logarithmic y axis allows for direct comparison with the time-dependant Nyquist limited Fourier transform analysis (figure 7).

coupled with the reference cell being the initial cell of the bin.

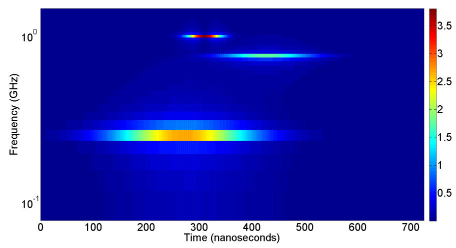

Figure 7: Time-dependant Nyquist limited Fourier transform analysis of the test trace in figure 5.

Figure 8 demonstrates the use of the analysis integrating the Nyquist limited technique, hence effectively matching bin sizes and frequencies in order to map the entire frequency range on a single graph. In comparison with Mead *et. al.*, the results show two distinct modes of frequencies in bands of 80-84 MHz and 72-76 MHz. These results differ from their experimental values of 63 MHz and 59 MHz, which were attributed to the north-south and either the east-west or vertical modes respectively [2].

A comparison of these bands with the standard Fourier transform of the dataset yields 2 corresponding frequency peaks at 82 MHz and 75 MHz, which are in good agreement with the theoretical model of 76 MHz and 82 MHz, belonging to the east-west and vertical E field modes respectively [2]. These modes are not immediately noticeable in the standard Fourier transform, as they are masked by the initial interaction noise. With this new method, we are now able to observe that the modes occurred 55 ns and 105 ns after the laser interaction; this is a new development in understanding that would not have been possible to ascertain without the time-dependant Nyquist limited Fourier transform analysis.

As the analysis can only compute up to the last full bin, detailed analysis of this section is lost. Investiga-

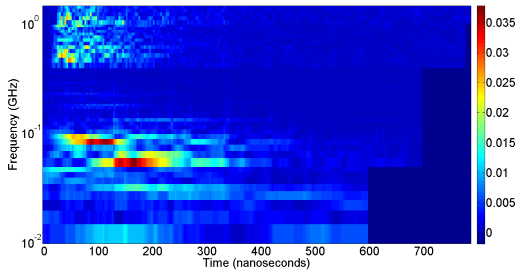

Figure 8: Time-dependant Nyquist limited Fourier transform analysis of the experimental EMP data of figure 2. The padding of the shorter arrays can be seen on the right of the figure, and more prominently in the lower frequencies. The 5% magnitude value was chosen in order to highlight that this region is not analysed data, whilst not disrupting the colourbar scales.

tions were undertaken to pad the original sample data with zeros of length equal to the largest bin; this would enable the analysis to conserve bin size and run until the start cell is the last the last recorded point, rather than the last point minus bin size that it is currently. This was not practical, as the pre-padding leads to artificially strong frequency components. This detrimentally impacts the colour-bar scaling, causing the analysis of the pre-padding data to become diluted in contrast.

5 Conclusion

In conclusion the time-dependant Nyquist limited Fourier transform analysis is capable of determining frequency as a function of time in the role of EMP analysis from the petawatt target chamber, showing that the frequency is delayed in generation. However, the technique is not limited to voltage time traces from oscilloscope readings, and the principals can be applied to any time base data sets from which a frequency analysis as a function of time would be beneficial.

6 Acknowledgements

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References

[1] C.N. Danson, P.A. Brummitt, R.J. Clarke, J.L.Collier, B. Fell, A.J. Frackiewicz, S. Hancock, S. Hawkes, C. Hernandez-Gomez, P. Holligan, M.H.R. Hutchinson, A. Kidd, W.J. Lester, I.O. Musgrave, D. Neely, D.R. Neville, P.A. Norreys, D.A. Pepler, C.J. Reason, W. Shaikh, T.B. Winstone, R.W.W. Wyatt and B.E. Wyborn. *Vulcan petawatt - An ultra-high-intensity interaction facility.* Nuclear Fusion **44** (2004) pp.S239-S246.

[2] M.J. Mead, D. Neely, J. Gauoin, R. Heathcote and P. Patel. *Electromagnetic pulse generation within a petawatt laser target chamber.* Review of Scientific Instruments **75** (2004) pp.4225-4227.

[3] P.H. Duncan *Analysis of the Moebius Loop Magnetic Field Sensor.* IEEE Transactions on Electromagnetic Compatibility $\boldsymbol{EMC-16}$ (1974) pp.73-89.

[4] C.E. Shannon *Communication in the Presence of Noise.* Proceedings of the IEEE **86** (1998) pp.447-457.

7 Appendix

Software used was MATLAB ® R2013b.

Average target type shot was 100μm Au.

The laser conditions were 600 J in 0.7 ps focused down to a 5 μm spot.

A moebius loop is a detector compromised of an electrical resistor that causes no magnetic interference. It is compromised of two conductive plates or wires separated by dielectric material that has been twisted through an angle of π radians and connected at either end to form a moebius loop. Further details in reference [3].

The moebius loop was located in the horizontal plane of the target, outside a central window port on the south wall of the chamber.