Light-by-Light Scattering at Low Energies

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Abstract

We consider the scattering of light by light using the Heisenberg-Euler low energy approximation. The target is taken to be a classical background field given by a high-intensity laser. We determine the scattering amplitude for a laser target modelled by (i) infinite and (ii) pulsed plane waves.

1 Introduction

High power lasers provide a unique source of photons. Current record intensities of 10^{22} W/cm^2 [1] will soon be exceeded by at least an order of magnitude at facilities such as the Extreme Light Infrastructure (ELI) [2] or, possibly, the Vulcan 20 PW upgrade project [3] at the RAL Central Laser Facility (CLF). The associated photon densities will then be in excess of $10^{18}/\mu$m^3. Put differently, in the diffraction limit the focus of a multi-Petawatt high-intensity laser will contain more than 10^{18} photons. In view of these huge photon densities it seems worthwhile to reconsider the options for an observation of photon-photon scattering.

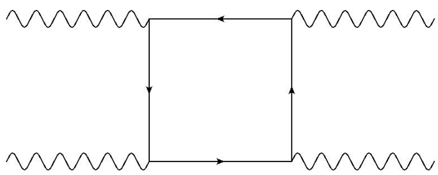

Figure 1: Feynman diagram for light-by-light or photon-photon scattering.

The possibility of this process, depicted in Fig. 1, was originally predicted in the early days of quantum electrodynamics (QED) by Halpern [4] as well as Heisenberg's school [5, 6]. For photon energies in the MeV regime or above, the cross section is of the order of 10^{-30} cm^2 [7], hence not too small by QED standards. However, in this regime, available photon fluxes are insufficient to observe any scattering. On the other hand, in the optical regime, where $\nu := \hbar\omega/mc^2 \ll 1$ (ω and m denoting laser frequency and electron mass, respectively) the cross section (in the centre-of-mass frame) is tiny [8],

$$\sigma_{\gamma\gamma} = \frac{973}{10125\,\pi^2}\alpha^2 r_e^2 \nu^6 \simeq 10^{-67} \text{ cm}^2 \; . \tag{1}$$

Note that the classical electron radius r_e is given by the product of the fine structure constant, $\alpha \simeq 1/137$ and the electron Compton wavelength, $\lambda_e = \hbar/mc$, so the cross section is indeed of order α^4 as the counting of vertices in Fig. 1 suggests.

In this paper we want to study the possibility of observing photon-photon scattering utilising the large photon densities available once multi-PW lasers go online. Compared to previous work [11] we will more closely adhere to a scattering picture as developed in [12, 13]. To this end, we will assume that two of the photons of Fig. 1 are stemming from an intense laser background, hence we will evaluate the modified Feynman diagram of Fig. 2.

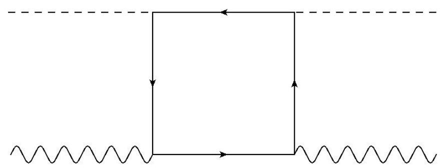

Figure 2: Feynman diagram showing the scattering of probe photons (wavy lines) off a classical laser background field (dashed lines).

To set the stage let us first briefly consider a more familiar situation, the scattering of charged particles off a static charge distribution described by a classical external potential $A_\mu(x)$. Following e.g. the text [10] the associated amplitude may be written as

$$T_{\text{fi}} = -i \int d^4x j^\mu_{\text{fi}}(x) A_\mu(x) \; , \tag{2}$$

with a transition current j^μ_{fi} built from initial and final plane wave states. For scalar QED the current is

$$j^\mu_{\text{fi}} = -ie(\phi^*_{\text{f}}\partial^\mu\phi_{\text{i}} - \phi_{\text{i}}\partial^\mu\phi^*_{\text{f}}) = -e(p^\mu_{\text{f}} + p^\mu_{\text{i}})e^{-iq\cdot x} \; , \tag{3}$$

where $q = p_{\text{f}} - p_{\text{i}}$ denotes the momentum transfer. Thus, the amplitude (2) becomes

$$T_{\text{fi}} = ie(p^\mu_{\text{f}} + p^\mu_{\text{i}}) \int d^4x \, e^{-iq\cdot x} A_\mu(x) = ie(p^\mu_{\text{f}} + p^\mu_{\text{i}}) A_\mu(q) \; , \tag{4}$$

and is thus given by the Fourier transform $A_\mu(q)$ of the potential times the vertex factor $ie(p^\mu_{\text{f}} + p^\mu_{\text{i}})$.

For spinor QED, the current is a spinor bilinear,

$$j^\mu_{\text{fi}} = -e\bar{u}_{\text{f}}\gamma^\mu u_{\text{i}} e^{-iq\cdot x} \; , \tag{5}$$

so that the amplitude becomes

$$T_{\rm fi} = ie\bar{u}_{\rm f}\gamma^\mu u_{\rm i} A_\mu(q) \ . \qquad (6)$$

If A_μ obeys the inhomogeneous wave equation with an external source J_μ, $\Box A_\mu = J_\mu$, its Fourier transform will be $A_\mu(q) = -J_\mu(q)/q^2$. For a static, spinless charge distribution, $J_\mu(x) = (\rho(\boldsymbol{x}), \boldsymbol{0})$. Defining the form factor as the (3d) Fourier transform of the charge distribution,

$$F(\boldsymbol{q}) := \int d^3x \, e^{i\boldsymbol{q}\cdot\boldsymbol{x}} \rho(\boldsymbol{x}) \ , \qquad (7)$$

the only non-vanishing current component is $J^0(q) = 2\pi\delta(q^0)F(\boldsymbol{q})$ and the amplitude (6) becomes

$$T_{\rm fi} = 2\pi i \, e u_{\rm f}^\dagger u_{\rm i} \, \delta(q^0) F(\boldsymbol{q})/\boldsymbol{q}^2 \ . \qquad (8)$$

In what follows we want to extend these considerations to the situation depicted in Fig. 2, i.e. when probe photons scatter off a classical electromagnetic field configuration describing a high-intensity laser. Both laser and probe energies will be assume small compared to mc^2.

2 Heisenberg-Euler Analysis

We employ the Heisenberg-Euler Lagrangian [6] in the leading order form already studied by Euler and Kockel [5],

$$\mathcal{L} = \mathscr{S} + \frac{1}{2}\big(c_- \mathscr{S}^2 + c_+ \mathscr{P}^2\big) \ . \qquad (9)$$

It depends only on the field invariants

$$\mathscr{S} = -\frac{1}{4} F_{\mu\nu}F^{\mu\nu} \ , \quad \mathscr{P} = -\frac{1}{4} F_{\mu\nu}\tilde{F}^{\mu\nu} \ , \qquad (10)$$

with low energy constants $c_\pm$ given by

$$\left\{ \begin{array}{c} c_+ \\ c_- \end{array} \right\} = \frac{\alpha}{45\pi} \frac{1}{E_S^2} \left\{ \begin{array}{c} 7 \\ 4 \end{array} \right\} \ , \qquad (11)$$

As usual, $E_S = m^2c^3/e\hbar \simeq 1.3 \times 10^{18}$ V/m is the QED electric field [6, 14, 15]. We assume probing a laser background described by field strength $F_{\mu\nu}$ with photons $f_{\mu\nu}$, so we split $F_{\mu\nu} \to F_{\mu\nu} + f_{\mu\nu}$ everywhere in (9). We denote the amplitude for photons with momentum l and polarisation ε ($\varepsilon \cdot l = 0$) to scatter to l' and ε', where $\varepsilon' \cdot l' = 0$, by

$$T_{\rm fi} = \langle \varepsilon', l' | S | \varepsilon, l \rangle \ , \qquad (12)$$

and write it akin to (4) and (6) as

$$T_{\rm fi} = -i\varepsilon_\mu l_\alpha V^{\mu\alpha,\nu\beta}(q) \varepsilon'_\nu l'_\beta \ . \qquad (13)$$

Here we have introduced momentum transfer $q = l' - l$ and a 'vertex function',

$$V^{\mu\alpha,\nu\beta}(q) = \int d^4x \, e^{-iq\cdot x} \, V^{\mu\alpha,\nu\beta}(x) \ , \qquad (14)$$

$$V^{\mu\alpha,\nu\beta}(x) = c_- F^{\mu\alpha}F^{\nu\beta} + c_+ \tilde{F}^{\mu\alpha}\tilde{F}^{\nu\beta} \ . \qquad (15)$$

Being quadratic in F, $V^{\mu\alpha,\nu\beta}(q)$ is basically a tensorial Fourier transform of the background *intensity*. The expression (13) represents the coupling of the tensor transition current, $\varepsilon'_\nu l'_\beta \varepsilon_\mu l_\alpha$, to the tensor vertex $V^{\mu\alpha,\nu\beta}$. Note that asymptotic polarisation and momentum vectors are treated on the same footing.

Contracting with the external momenta l, l', one defines the polarisation tensor

$$\Pi^{\mu\nu}_{ll'}(q) \equiv l_\alpha \tilde{V}^{\mu\alpha,\nu\beta}(q) l'_\beta \ , \quad q = l' - l \ , \qquad (16)$$

whereupon the amplitude (13) takes on the simple form

$$T_{\rm fi} = -i\varepsilon_\mu \Pi^{\mu\nu}_{ll'}(q) \varepsilon'_\nu \ . \qquad (17)$$

Specialising to forward scattering where $q = 0$ leads to the forward scattering amplitude

$$T_{\rm fi,fwd} = -i\varepsilon_\mu \Pi^{\mu\nu}_{l}(0) \varepsilon'_\nu \ . \qquad (18)$$

To make contact with some of the literature employing the polarisation tensor [16, 17, 18] we define the 4-vectors

$$b^\mu_l(x) := F^{\mu\nu}(x) l_\nu \ , \ \tilde{b}^\mu_l(x) := \tilde{F}^{\mu\nu}(x) l_\nu \ . \qquad (19)$$

As a result, the off-forward polarisation tensor $\Pi^{\mu\nu}_{ll'}(q)$ is the Fourier transform of an off-diagonal spectral representation,

$$\Pi^{\mu\nu}_{ll'}(q) = \int d^4x \, e^{-iq\cdot x} \, (c_- b^\mu_l b^\nu_{l'} + c_+ \tilde{b}^\mu_l \tilde{b}^\nu_{l'}) \ . \qquad (20)$$

In the forward limit, $q = 0$, this turns into the Fourier zero mode

$$\Pi^{\mu\nu}_l(q = 0) = \int d^4x \, \Pi^{\mu\nu}_l(x) \ , \qquad (21)$$

of the diagonal ($l = l'$) position space spectral representation

$$\Pi^{\mu\nu}_l(x) = c_- b^\mu_l b^\nu_l + c_+ \tilde{b}^\mu_l \tilde{b}^\nu_l \ . \qquad (22)$$

The dual field strengths $\tilde{F}$ in the above may be traded for the Maxwell energy momentum tensor,

$$\Theta^{\mu\nu} = F^{\mu\alpha} F_\alpha{}^\nu - g^{\mu\nu} \mathscr{S} \qquad (23)$$

via the useful relation

$$F^{\mu\alpha}F^{\nu\beta} + \tilde{F}^{\mu\alpha}\tilde{F}^{\nu\beta} = - g^{\mu\nu}\Theta^{\alpha\beta} - g^{\alpha\beta}\Theta^{\mu\nu} + g^{\alpha\nu}\Theta^{\mu\beta} + g^{\beta\mu}\Theta^{\alpha\nu} \ , \qquad (24)$$

which follows by evaluating the two Levi-Civita tensors in the $\tilde{F}\tilde{F}$ term by means of a determinant. Plugging this into (15) the polarisation tensor (16) in position space becomes ($\Delta c \equiv c_+ - c_-$),

$$\Pi^{\mu\nu}_{ll'}(x) = -\Delta c \, b^\mu_l b^\nu_{l'} - c_+ \Big\{ g^{\mu\nu}(l, \Theta l') - l \cdot l' \Theta^{\mu\nu} + l^\nu \Theta^{\mu\alpha} l'_\alpha + l'^\mu \Theta^{\nu\alpha} l_\alpha \Big\} \ . \qquad (25)$$

Its forward limit is obtained by setting $l = l'$ and $l \cdot l' = l^2 = 0$,

$$\Pi_l^{\mu\nu} = -\Delta c\, b^\mu b^\nu - c_+ \Big\{ g^{\mu\nu}(l, \Theta l) + l^\nu \Theta^{\mu\alpha} l_\alpha + l^\mu \Theta^{\nu\alpha} l_\alpha \Big\} \,, \tag{26}$$

which, according to (18), leads to the forward scattering amplitude

$$T_{\text{fi,fwd}} = -i \int d^4x \, (-\Delta c \, \varepsilon \cdot b \; \varepsilon' \cdot b + c_+ \varepsilon \cdot \varepsilon' \, b^2) \,. \tag{27}$$

Note that all the background dependence resides in the eigenvector $b(x)$ of the polarisation tensor. The first term, proportional to the difference Δc of $\mathscr{S}$ and $\mathscr{P}$ terms in the Lagrangian, measures the relative polarisations of probe and background while the second term measures the relative polarisation of incoming and outgoing probe polarisations independent of the background. For the Born-Infeld Lagrangian one has $\Delta c = 0$ and only this latter term contributes [17, 19].

Hence, the first term in (27) is the amplitude for the polarisation to flip into an orthogonal state with $\varepsilon \cdot \varepsilon' = 0$,

$$T_{\text{fi,fwd,flip}} = i \Delta c \int d^4x \, \varepsilon \cdot b \; \varepsilon' \cdot b \,. \tag{28}$$

The square of this amplitude directly measures vacuum birefringence [12, 13]. For Born-Infeld theory the signal (28) is identically zero, hence there is no vacuum birefringence, in accordance with [17].

3 Plane Wave Backgrounds

Let us evaluate the scattering amplitude for a laser background modelled by a plane wave. In this case both ordinary and dual field strengths are null and may be written in terms of a dreibein consisting of a null vector n and two polarisation vectors ϵ_1 and ϵ_2 (adopting linear polarisation) with $n \cdot \epsilon_i = 0 = \epsilon_1 \cdot \epsilon_2$,

$$F^{\mu\nu}(x) = F(x)(n^\mu \epsilon_1^\nu - n^\nu \epsilon_1^\mu) \,, \tag{29}$$

$$\tilde{F}^{\mu\nu}(x) = F(x)(n^\mu \epsilon_2^\nu - n^\nu \epsilon_2^\mu) \,. \tag{30}$$

For a plane wave, the profile function F only depends on the phase $k \cdot x$, $F(x) = F(k \cdot x)$, where $k^\mu = (\omega/c) n^\mu$ denotes the wave vector. F will be chosen appropriately below, but for the time being we leave it arbitrary noting that its square corresponds to an energy density, $F^2 =: w$ (or intensity divided by c).

Let us evaluate the scattering amplitude from (12), $\langle \varepsilon', l' | S | \varepsilon, l \rangle$, assuming a head-on collision where $l \cdot \epsilon_i = 0$. In this case the polarisation tensor (25) can be decomposed as

$$\Pi_{ll'}^{\mu\nu}(x) =: w(x) w_{ll'}^{\mu\nu} \,, \tag{31}$$

with the intensity profile $w(x)$ multiplying the constant tensor

$$w_{ll'}^{\mu\nu} := n \cdot l \Big\{ c_- \Big(n \cdot l' \, \epsilon_1^\mu \epsilon_1^\nu - \epsilon_1 \cdot l' \, \epsilon_1^\mu n^\nu \Big) + c_+ \, (1 \leftrightarrow 2) \Big\} \,. \tag{32}$$

Contracting with the probe polarisation vectors, the scattering amplitude becomes

$$\langle \varepsilon', l' | S | \varepsilon, l \rangle = -i \varepsilon_\mu w_{ll'}^{\mu\nu} \varepsilon'_\nu \, w(q) \,. \tag{33}$$

On the right-hand side, the function $w(q)$ represents the Fourier transform of the intensity profile $F^2(x)$ and thus is rather similar to a form factor. We will call it the intensity form factor henceforth. It contains all the information about the 'shape' of the background intensity distribution (i.e. the target laser focus). The prefactor, $\varepsilon w \varepsilon'$, carries the information about probe and target polarisations. It becomes particularly simple for forward scattering, $l = l'$, for which we obtain

$$\varepsilon_\mu w_{ll'}^{\mu\nu} \varepsilon'_\nu = (n \cdot l)^2 \, \varepsilon_\mu \Big(c_- \epsilon_1^\mu \epsilon_1^\nu + c_+ \epsilon_2^\mu \epsilon_2^\nu \Big) \varepsilon'_\nu \,. \tag{34}$$

The form (27) is recovered by eliminating $\epsilon_2^\mu \epsilon_2^\nu$ by means of the completeness relation

$$g^{\mu\nu} = \frac{1}{2} n^\mu n^\nu + \frac{1}{2} \bar{n}^\mu \bar{n}^\nu - \epsilon_1^\mu \epsilon_1^\nu - \epsilon_2^\mu \epsilon_2^\nu \,. \tag{35}$$

Here we have introduced a second null vector $\bar{n}$ with $n \cdot \bar{n} = 2$ and $\bar{n} \cdot \epsilon_i = 0$. For a head-on collision $\varepsilon \cdot \bar{n} = 0$ while forward scattering implies $\varepsilon' \cdot \bar{n} = 0$. As a result, in (34) one can make the replacement

$$\epsilon_2^\mu \epsilon_2^\nu \to -g^{\mu\nu} - \epsilon_1^\mu \epsilon_1^\nu \,, \tag{36}$$

so that we obtain

$$\varepsilon_\mu w_{ll'}^{\mu\nu} \varepsilon'_\nu = -(n \cdot l)^2 \, (\Delta c \, \varepsilon \cdot \epsilon_1 \; \varepsilon' \cdot \epsilon_1 + c_+ \, \varepsilon \cdot \varepsilon') \,, \tag{37}$$

which are indeed the terms present in (27) if one notes that

$$b^\mu = -F n \cdot l \; \epsilon_1^\mu \,, \tag{38}$$

$$\tilde{b}^\mu = -F n \cdot l \; \epsilon_2^\mu \,. \tag{39}$$

Hence, the eigenvectors of the forward polarisation tensor are indeed proportional to the background polarisation vectors, ϵ_i^μ.

As the polarisation and shape dependence factorises according to (33), different plane wave laser backgrounds, given their polarisation state, only differ in their shape. To compare the latter, it is hence sufficient to consider only the shape form factors. In what follows we will briefly discuss two examples, infinite and pulsed plane waves.

3.1 Infinite Plane Wave

The simplest model for the target laser beam/focus is an infinite, monochromatic plane wave, given by the shape function

$$F(x) = F(k \cdot x) = F_0 \sin(k \cdot x) \; , \qquad (40)$$

where F_0 denotes the amplitude. Clearly, in reality there is no such thing as an infinite plane wave, but we can expect (40) to be a reasonably good model for lasers that are not too strongly focussed and being probed close to the optical axis. A straightforward Fourier transform of F^2 then yields the intensity form factor

$$w(q) = -\frac{(2\pi)^4}{4} F_0^2 \left\{ \delta^4(q - 2k) - 2\delta^4(q) + \delta^4(q + 2k) \right\} \; . \qquad (41)$$

Thus, the the momentum transfer can be $\pm 2k$ or zero, the latter of course corresponding to forward scattering. This has been observed before [20] and corresponds to the three diagrams in Fig. 3.

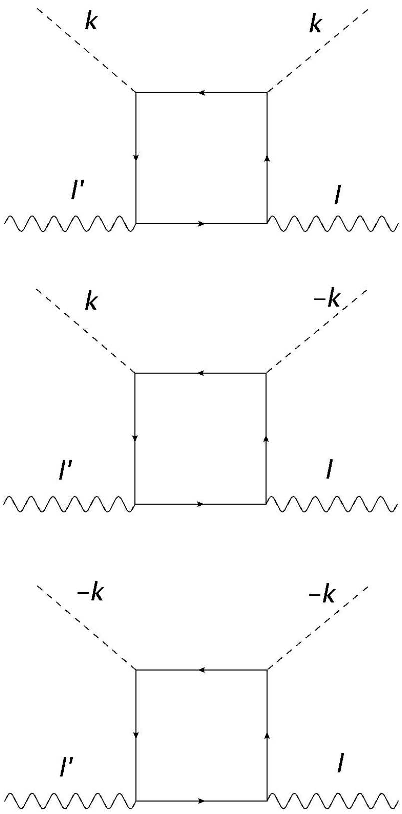

Figure 3: The three Feynman diagrams showing the possible momentum transfer experienced by probe photons (wavy lines) scattering off a laser background (dashed lines). From top to bottom, the laser momentum signs correspond to: two absorptions ($l' - l = 2k$), an absorption and an emission ($l' - l = 0$, forward scattering) and two emissions ($l' - l = -2k$).

To evaluate the delta functions in (41) a bit further we assume (without loss of generality) that the target laser is propagating along the z axis. We can then rewrite the phase $k \cdot x$ according to

$$k \cdot x = (\omega/c) n \cdot x =: (\omega/c) x^- = k^+ x^- / 2 \; , \qquad (42)$$

introducing the light front coordinate $x^- := n \cdot x = ct - z$, Fourier conjugate to the longitudinal momentum component, $k^+ := k^0 + k^3 = \omega/c + k_z$. Defining a 3d delta function

$$\delta^3(\mathsf{q}) := \frac{1}{2} \delta(q^-) \delta^2(\boldsymbol{q}_\perp) \; , \quad \boldsymbol{q}_\perp := (q_x, q_y) \; , \qquad (43)$$

the prefactor 1/2 stemming from the lightfront metric, cf. $d^4x = (1/2) dx^+ dx^- d^2x_\perp$, we can rewrite (41) as

$$w(q) = -\frac{(2\pi)^4}{4} F_0^2 \, \delta^3(\mathsf{q}) \sum_{s=0,\pm 2} \delta(q^+ + s k^+) \; . \qquad (44)$$

Thus, the three momentum components l^-, $\boldsymbol{l}_\perp$ are strictly conserved while l^+ is either conserved (forward scattering) or changes by the fixed amounts $\pm 2k^+$.

3.2 Pulsed Plane Wave

A somewhat more realistic model is provided by a pulsed plane wave which has finite extent in the phase variable $k \cdot x$. For the sake of definiteness we choose a Gaussian envelope of width Δ,

$$F(k \cdot x) = F_0 \sin(k \cdot x) \, \exp\left\{ -(k \cdot x)^2 / 2\Delta^2 \right\} \; . \qquad (45)$$

Note that the background still depends on $k \cdot x$ only as is appropriate for a plane wave. Fourier transforming F^2 as before again results in three terms,

$$w(q) = \sum_{s=0,\pm 2} w_0(q + sk) \; , \qquad (46)$$

where w_0 represents the Fourier transform of the squared Gaussian in (45),

$$\begin{aligned} w_0(q) &:= -\frac{1}{4} F_0^2 \int d^4x \, \exp\left\{ -(k \cdot x)^2/\Delta^2 - iq \cdot x \right\} \\ &= -\frac{(2\pi)^4}{4} F_0^2 \, \delta^3(\mathsf{q}) \sqrt{\frac{\Delta}{4\pi\omega}} \exp\left\{ -\left(\frac{\Delta}{4\omega} q^+ \right)^2 \right\} \; . \end{aligned} \qquad (47)$$

Comparing with (41) and (44) we note that the light front momentum transfer q^+ is no longer determined sharply. This is because the intensity form factor $w(q)$ now contains a range of Fourier modes (and is not just supported at $q^+ = sk^+$, $s = 0, \pm 2$). However, for large pulse width Δ the form factor $w(q)$ will be strongly peaked at $q^+ = sk^+$. In the infinite width limit, $\Delta \to \infty$,

$$\sqrt{\frac{\Delta}{4\pi\omega}} \exp\left\{ -\left(\frac{\Delta}{4\omega} q^+ \right)^2 \right\} \to \delta(q^+) \; , \qquad (48)$$

and we do recover (41) as we must.

4 Conclusion

We have seen that interactions between probe photons and intense laser beams are possible due to the nonlinearities inherent in QED. For low energy probes (optical or X-ray beams) the nonlinearities are made explicit in terms of the Heisenberg-Euler Lagrangian. Based on the latter one can adopt a straightforward scattering picture which describes what may be termed 'nonlinear vacuum optics'. In the above, we have calculated the light-by-light scattering amplitude relevant for photon-laser interactions. Interestingly, the amplitude factorises in polarisation and shape dependent terms, the latter given by the intensity form factor, i.e. the Fourier transform of the target intensity profile. The next steps are to consider more realistic backgrounds such as Gaussian beams (see [12, 13] for recent exploratory work on birefringence along these lines). As Gaussian beams contain (small) longitudinal field components, it is also possible to have effects such as photon emission from the vacuum (three-wave mixing) [21, 22]. In this case, the scattering amplitude contains terms of the form $(FF)(Ff)$ which vanish for the plane waves discussed in this contribution. To maximise the amplitude one hence needs to abandon plane waves and consider ultra-strong focussing.

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