Characterising the Acceleration Phase of Blast Wave Formation

Contact: tef503@york.ac.uk

T. E. Fox
York Plasma Institute,
University of York,
YO10 5DD, United Kingdom

A. P. L. Robinson
Central Laser Facility,
STFC Rutherford Appleton Laboratory,
OX11 0QX, United Kingdom

H. Schmitz
Central Laser Facility,
STFC Rutherford Appleton Laboratory,
OX11 0QX, United Kingdom

J. Pasley
York Plasma Institute,
University of York,
YO10 5DD, United Kingdom

Abstract

Intensely heated, localised regions in uniform fluids will rapidly expand and generate an outwardly propagating blast wave. The Sedov-Taylor self-similar solution for such blast waves has long been studied and applied to a variety of scenarios. A characteristic time for their formation has also long been identified using dimensional analysis, which by its very nature, can offer several interpretations. We propose that, rather than simply being a characteristic time, it may be interpreted as the definitive time taken for a blast wave resulting from an intense explosion in a uniform media to contain its maximum kinetic energy. A scaling relation for this measure of the acceleration phase, preceding the establishment of the blast wave, is presented and confirmed using a 1D planar hydrodynamic model.

1 Introduction

The structure of a strong shock wave formed by an intense explosion in a fluid has a self-similar solution [Taylor(1950a), Sedov(1959)] that has been applied to diverse scenarios including chemical and nuclear explosions [Taylor(1950b)], supernova evolution [Chevalier(1976)] and expanding laser-plasmas [Leonard and J.(1975)]. The well known scaling relation for position of the shock at late times can be derived using dimensional analysis without reference to the self-similar solution. From this relation it is clear both the scaling relation and the self-similar solution describe only the phase of evolution in which the shock is gradually decelerating. The early explosive phase is not described by the Sedov-Taylor solution.

A characteristic time for the evolution of a blast wave solution from a finite region of internal energy has been identified via dimensional analysis [Zel'dovich and Raizer(1966), Gull(1973)]. By the nature of dimensional analysis, this characteristic time can offer several interpretations but will always have the same dependence on physical parameters. We propose that, rather than simply being a characteristic time, it may be interpreted as the definitive time taken for an initial, localised source of internal energy to drive a blast wave containing its maximum amount of kinetic energy. Such a measure can be of some utility if one is interested in the timescale of the initial acceleration phase of blast wave formation. For example, in some aspects of laser fusion and laser-driven hydrodynamics one may be more interested in the early acceleration phase rather than the Sedov-Taylor phase that sets in much later.

In this paper we study the formation of Sedov-Taylor-like blast waves due to a localised high pressure region in an otherwise uniform, cold fluid. We present a scaling relation for the time taken for that maximum to be achieved and compare the resultant blast wave to the Sedov-Taylor self-similar solution. In Section 2, we develop the scaling relation using dimensional analysis. In Section 3, we confirm the scaling relation using a 1D planar hydrodynamic code and present our concluding remarks in Section 4.

2 Theory

A spatially localised heated region within an otherwise uniform cold fluid will expand and drive hydrodynamic waves into the surrounding fluid. When the pressure difference between the hot and cold regions is significant, shock waves will be generated. The thermal energy of the heated region is converted into kinetic energy as the shock wave develops. The Sedov-Taylor self-similar solution describes the evolution of an established blast wave and predicts the final energy partition between thermal and kinetic energy, but does not describe the formation of the blast wave when the fluid is being accelerated by the initial thermal pressure. This can be seen from the Sedov-Taylor relation for the position of the shock front:

$$R_s \propto \left(\frac{E}{\rho}\right)^{1/5} t^{2/5}, \qquad (1)$$

where E is the thermal energy in the initial hot spot, and ρ is the ambient density. We can physically interpret this if we differentiate this expression to obtain, $\dot{R}_s \propto t^{-3/5}$,

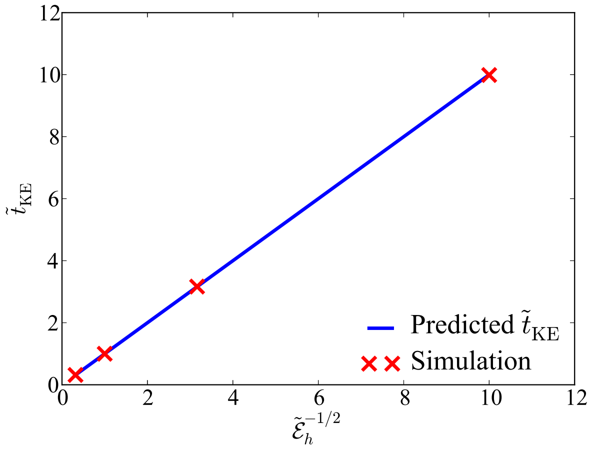

Figure 1: The scaling of $\tilde{t}_{\rm KE}$ with $\tilde{\mathcal{E}}_h$, as predicted by Eq. 3 is confirmed by varying $\tilde{\mathcal{E}}_h$ while keeping $\tilde{\rho}_0$ and $\tilde{R}_h$ constant.

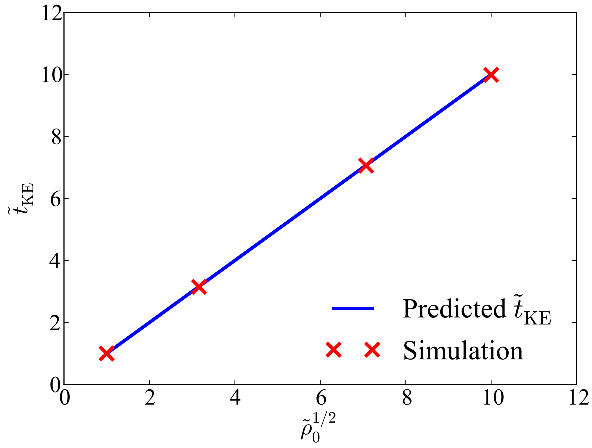

Figure 2: The scaling of $\tilde{t}_{\rm KE}$ with $\tilde{\rho}_0$, as predicted by Eq. 3 is confirmed by varying $\tilde{\rho}_0$ while keeping $\tilde{\mathcal{E}}_h$ and $\tilde{R}_h$ constant.

i.e. the shock front velocity is falling with time. Therefore the Sedov-Taylor solution can only describe the behaviour at late time, and it doesn't describe the initial explosive or acceleration phase.

Now let us consider applying dimensional analysis to the acceleration phase. Let the duration of this acceleration phase be $t_{\rm KE}$, that is, the time taken for the maximum possible conversion from thermal to kinetic energy to be achieved. We consider this to depend on the characteristic spatial size R_h of the heated region, the initial internal energy E_h contained within the region and the ambient density of the cold fluid ρ_0. Using these parameters one can construct a dimensionless parameter η. Then the time taken for the forming blast wave to contain its maximum kinetic energy obeys the scaling relation:

$$t_{\rm KE} = \eta\sqrt{\frac{\rho_0}{E_h}}R_h^{5/2}. \tag{2}$$

Equation 2 is necessarily of the same form as the Sedov-Taylor scaling relation but its application, or physical interpretation, is fundamentally different. This scaling relation is validated in Sec. 3 using a 1D planar hydrodynamic model. In a planar geometry it is meaningful to replace the initial thermal energy E_h with the areal thermal energy $\mathcal{E}_h$. The scaling relation is then:

$$t_{\rm KE} = \xi\sqrt{\frac{\rho}{\mathcal{E}_h}}R_h^{3/2}, \tag{3}$$

where ξ is the dimensionless parameter in the planar case.

3 Hydrodynamic Modelling

The scaling relation in Eq. 3 is confirmed by way of simple 1D planar, single fluid hydrodynamic simulations. The model solves the Euler equations in conservative form using the algorithm put forward by Ziegler [Ziegler(2004)], and has been applied to standard shock tube problems [Sod(1978)] with excellent agreement.

The simulations are initialised with a uniform fluid density ρ_0, containing a Gaussian shaped heated layer at its centre. The characteristic width of the heated layer is R_h and is equal to its full-width-at-half-maximum (FWHM). This width of fluid contains the initial areal thermal energy $\mathcal{E}_h$. Shock wave formation in a cold plasma is approximated by setting the ratio of the maximum and minimum initial areal energy to be at least $\sim 10^{13}$ in all simulations. An ideal gas is assumed with $\gamma = 5/3$, unless otherwise stated.

Convergence testing led to the choice of simulation parameters. A spatial resolution of $R_h^{\rm min}/\Delta x = 25$ was used in all simulations, where $R_h^{\rm min}$ is the minimum characteristic width of the heated layer used in the investigation. Doubling the resolution changed the measured t_{KE} by $\sim$1%, and the maximum areal kinetic energy measured by $\sim$0.3%. As long as the time step satisfied the usual hydrodynamic stability condition, the measured values did not vary with temporal resolution.

Figures 1 to 3 show the measured times taken for the shock wave to develop its maximum amount of areal kinetic energy for a parameter scan of initial conditions. Their excellent agreement with Eq. 3 shows that the acceleration phase of blast wave evolution is well characterised. In these figures, $\tilde{t}_{\rm KE}$, $\tilde{\mathcal{E}}_h$, $\tilde{\rho}_0$ and $\tilde{R}_h$ are dimensionless quantities defined as the ratio of their dimensional equivalents relative to a reference simulation that exists in each parameter scan. The scaling of $\tilde{t}_{\rm KE}$ with $\tilde{\mathcal{E}}_h$, $\tilde{\rho}_0$ and $\tilde{R}_h$ is confirmed by holding two of the parameters constant while varying the third. In these units, the predicted curves for $\tilde{t}_{\rm KE}$ are plotted with $\xi = 1$. The arithmetic mean of the measured scaling parameters is

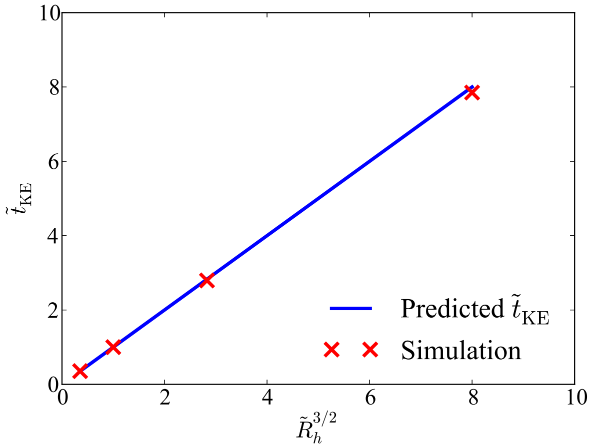

Figure 3: The scaling of $\tilde{t}_{\mathrm{KE}}$ with $\tilde{R}_h$, as predicted by Eq. 3 is confirmed by varying $\tilde{R}_h$ while keeping $\tilde{\rho}_0$ and $\tilde{\mathcal{E}}_h$ constant.

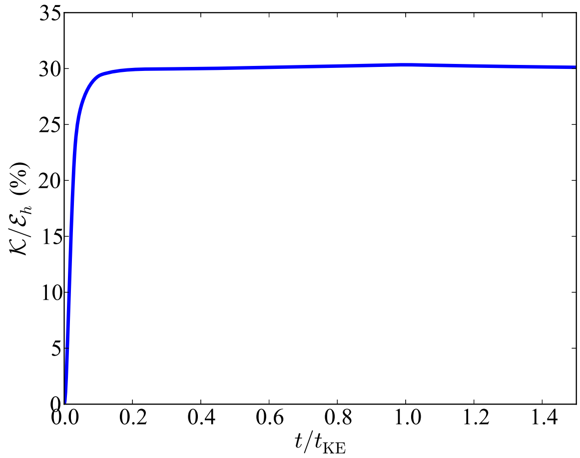

Figure 4: Evolution of the fraction of initial areal internal energy converted to areal kinetic energy as a function of time taken to achieve maximum conversion.

$\bar{\xi} = 0.998$ with a $\sim$2.5% spread between the maximum and minimum measured values of ξ.

The history of the conversion of initial areal internal energy $\mathcal{E}_h$ into areal kinetic energy $\mathcal{K}$ is shown in Fig. 4. There are two distinct phases in the rate of conversion: a rapid early conversion phase followed by a much more gradual late conversion phase. Looking at Fig. 4, one would be inclined to say that the acceleration phase ends at $\sim 0.2 t_{\mathrm{KE}}$. However, the acceleration phase strictly ends at t_{KE}, but this time lies well into the late conversion phase. The difference is that, although 98% of the total conversion has occurred by $0.17 t_{\mathrm{KE}}$, the remaining 2% of the total conversion takes considerably longer. The scaling relations in Eq. 3 still hold if one takes the measurement after 95%, 98% and 99% of the total conversion, corresponding to times of $0.09 t_{\mathrm{KE}}$, $0.17 t_{\mathrm{KE}}$, $0.54 t_{\mathrm{KE}}$, respectively, but with different values of ξ.

Figure 4 shows that the energy partition within the blast wave is 30% areal kinetic energy and therefore 70% areal thermal energy. This can be compared to the Sedov-Taylor self-similar solution for blast waves in a 1D planar geometry, which predicts a partition of 23% areal kinetic energy and 77% areal thermal energy. The difference between the simulations and the theory, as well as the late conversion phase evolution seen in Fig. 4, are due to the simulated blast wave not being exactly self-similar. This non-adherence to self-similarity is expected due to the initial conditions and has been observed in studies of supernova remnants [Gull(1973), Gull(1975), Cioffi, McKee, and Bertschinger(1988), Dohm-Palmer and Jones(1996)]. These studies report on the observance of reverse shocks and transient waves travelling back and forth along the blast wave profile as they reflect off each other at the origin and off the blast wave front. Eventually these will dissipate, allowing the blast wave to assume a self-similar form, however this can take a significant amount of time [Gull(1973), Wheeler, Mazurek, and Sivaramakrishnan(1980)]. Such transient waves exist for the duration of our simulations and, according to the equipartition theorem, should push the system closer to an equal partition of energy between thermal and kinetic energy, as observed. The spatial resolution was doubled but the same features are present, and increasing the resolution did not noticeably alter the kinetic energy conversion history. Changing the adiabatic constant to $\gamma = 7/5$ yields a 76% to 24% partition between thermal and kinetic energy, respectively, which is also in relatively good agreement with the self-similar solution which predicts a partition 82% and 18% between thermal and kinetic energy, respectively.

4 Conclusions

The propagation of blast waves and their relaxation to the Sedov-Taylor self-similar solution has received a lot of attention over the decades [Gull(1973), Gull(1975), Cioffi, McKee, and Bertschinger(1988), Dohm-Palmer and Jones(1996), Chevalier(1976), Leonard and J.(1975), Book(1994)] and the acceleration phase of their formation, preceding the Sedov-Taylor phase, has been described by a characteristic time [Zel'dovich and Raizer(1966), Gull(1973)]. The origins of this characteristic time mean that it is open to reinterpretation. We have demonstrated that, rather than simply being a characteristic time, it may be interpreted as the definitive time taken for an accelerating blast wave to contain its maximum kinetic energy. An understanding of the time taken for a shock wave to develop is of interest when it may be an appreciable fraction of the system timescale. Such a situation is in inertial confinement fusion where controlled generation of shock waves is of fundamental importance.

We have studied the case of blast wave formation resulting from an intense explosion in a uniform medium and have shown that the time taken for the blast wave to contain its maximum kinetic energy is governed by a simple scaling law. The evolution of the kinetic energy conversion consists of two distinct phases. Most of the kinetic energy is generated during the short, early conversion phase which sees the formation of a blast wave modulated by transient waves that take the solution away from self-similarity. The late conversion phase shows a much more gradual evolution in the partition of energy as the system begins to settle into the self-similar solution.

This 1D single-fluid, hydrodynamic study is limited in terms of the physics included but does give an insight into the basic scalings that are invoked during blast wave formation.

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