Weak collisionless shocks in laser-plasmas

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Abstract

We describe an analytic theory giving the structure of weak laminar collisionless shocks in a plasma and show that it may be of relevance to observations of very strong localized electric fields in laser compressed pellets, to recent experiments on ion acceleration and to species separation in ICF targets.

1 Introduction

Observations on laser compressed targets have shown the existence of very strong electric fields, of the order of 10^9 - 10^{10} V/m, localized over distances of the order of 100 nm [1–4]. Recently we have suggested that these might be readily explained as weak laminar collisionless shock structures [5], rather than dissipative shocks as suggested elsewhere [4]. Here, we shall show that such weak shocks can, indeed, give rise to fields of the observed magnitude and length scale. We shall also show that they can reproduce the main features of some recent experiments on ion acceleration [6] and, finally, discuss the possible implications for species separation in fusion targets.

2 Theory

If we consider a single ion species flowing into a region where the potential increases monotonically from zero to $\phi_{\max}$, in the rest frame of the shock, then the ion density in the upstream region, normalized to the density of the incoming flow, is

$$n_i(\phi,\phi_{\max}) = \frac{1}{\sqrt{2\pi}}\int_0^\infty \exp\left[-\frac{1}{2}\left(\sqrt{v^2+2\phi}-V\right)^2\right]dv + \frac{1}{\sqrt{2\pi}}\int_0^{\sqrt{2(\phi_{\max}-\phi)}} \exp\left[-\frac{1}{2}\left(\sqrt{v^2+2\phi}-V\right)^2\right]dv \quad (1)$$

with velocities normalized to the ion thermal velocity V_i and the potential ϕ to $m_i V_i^2/e$, assuming the ions to be singly charged. Here V is the incoming flow velocity, assumed to be well above the thermal velocity so that the Maxwellian thermal spread around this velocity has no significant backward flowing part. The first term is the density of ions flowing into the shock, while the second is the reflected ion component, upstream of the potential maximum. In the laminar shock analysis presented here ion reflection is the dominant dissipation mechanism rather than collisions. Reflection occurs due to the space charge effects resulting in an electric field equation (5) discussed below that is responsible for ion separation demonstrated in this paper.

For the electrons we make the assumption that on the time scales involved in the shock they take up a thermal distribution, so that the electron density is given by

$$n_e = n_0 \exp\left(\frac{\phi}{T}\right)$$

with T the ratio of the electron to ion temperatures. We also assume that the electrons flow so as to neutralize the ion charge far upstream where ϕ tends to zero, so that $n_0(\phi_{\max}) = n_i(0,\phi_{\max})$. It is convenient to introduce a Mach number in terms of the ion sound speed $c_s = \sqrt{T_e/m_i}$ or $\sqrt{T}$ in our normalized units. This neglects the contribution of the ion pressure to the sound speed. Since the structures that concern us only exist when the electron temperature is well in excess of the ion temperature this definition of the Mach number is never very far from the true Mach number.

We can now obtain Poisson's equation in the dimensionless form

$$\frac{d^2\phi}{dx^2} = \left[n_e(\phi,\phi_{\max}) - n_i(\phi,\phi_{\max})\right], \quad (2)$$

where the length scale is V_i/ω_{pi} with ω_{pi} the ion plasma frequency. To determine the value of $\phi_{\max}$ consistent with the dynamics of the system we introduce the Sagdeev potential (or pseudopotential) Ψ_s such that Eqn. (2) takes the form

$$\frac{d^2\phi}{dx^2} = -\frac{\partial\Psi_s(\phi,\phi_{\max})}{\partial\phi}. \quad (3)$$

and the problem becomes analogous to that of the motion of a particle in the Sagdeev potential. Our assumption of a monotonically increasing potential reaching a

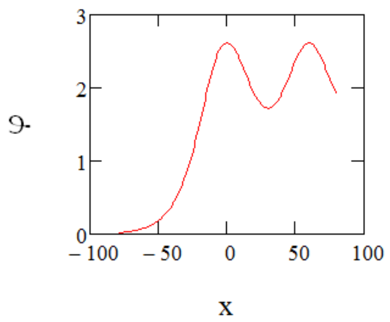

Figure 1: Profile of the pseudo-potential in normalized variables with $T = 20$ and $M = 1.15$. If we take the ion temperature as 100 eV then this gives a maximum electric field of 1.1×10^{10} V/m and a length scale of about 40 nm for the main potential ramp.

maximum at $\phi_{\max}$ requires that Ψ_s has a maximum at $\phi = 0$ (where we can take $\Psi_s = 0$) and returns to zero at $\phi = \phi_{\max}$, forming a potential well. This means that we must have

$$\Psi_s(\phi_{\max}, \phi_{\max}) = 0, \tag{4}$$

with $\Psi_s < 0$ in the interval $0 < \phi < \phi_{\max}$. The Sagdeev potential is too complicated to allow any analytic progress on this condition, but numerical investigation shows that the required behaviour only occurs within suitable ranges of the temperature ratio and Mach number. If the Sagdeev potential were the same in the downstream region we would find a solitary wave solution but the symmetry is destroyed by the absence of reflected ions there. To obtain the potential profile we find a solution to Eqn. (4), then integrate Poisson's equation upstream and downstream starting with the initial conditions $\phi = \phi_{\max}$ and $d\phi/dx = 0$. A representative solution is shown in Figure 1. The oscillatory profile downstream is typical of the behaviour we find, the amplitude of the downstream oscillations decreasing as the Mach number increases until the latter reaches a critical value beyond which a solution no longer exists. This is consistent with behaviour found by Forslund and Freidberg in computer simulations many years ago [7], where a laminar structure of the type found here, with only a few reflected ions, was found to go over to a more complex structure with almost all upstream ions reflected.

3 Fields in pellets

To relate our theory to the observations of localized fields in pellets mentioned in the introduction we must consider the relation between our normalized variables and physical quantities. The relevant relations for the electric field and the the length are

$$E[\mathrm{V/m}] = 4.26 \times 10^{-3} E_{\mathrm{norm}} (T_i[\mathrm{keV}])^{1/2} (n_i[\mathrm{m}^{-3}])^{1/2} \tag{5}$$

$$L[\mathrm{m}] = 2.35 \times 10^{5} L_{\mathrm{norm}} (T_i[\mathrm{keV}])^{1/2} (n_i[\mathrm{m}^{-3}])^{-1/2}.$$

Note that these only have a square root dependence on the ion temperature and density and that the ion mass does not appear. The square root behaviour means that the values obtained are not critically dependent on the assumed plasma parameters. For the example shown in Figure 1 we obtain, assuming an ion temperature of 100 eV and a density of 10^{28} m^{-3}, a peak electric field of 1.1×10^{10} V/m and a length scale of around 40 nm for the main potential ramp. This is in good agreeement with the observations. The constraints on the allowed Mach number for these laminar structures means that it is not possible to obtain values differing much from this by varying the parameters.

4 Ion acceleration

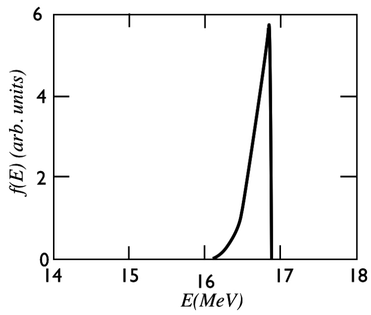

Figure 2: The energy spectrum of reflected ions for the parameters given in the text.

We now consider the results on ion acceleration of Haberberger *et al.* [6]. These show a very narrow energy spread, of about one percent of the ion energy, but a rather small number of ions in the beam. The acceleration is produced by having a series of laser pulses with the acceleration presumed to be generated by a shock launched into an expanding plasma produced by the initial pulses. Computer simulations given in this paper and elsewhere [8] show a much larger energy spread and are obtained from a shock of Mach number around 2. This is well above the limit for our laminar structures and in the regime where the early simulations of Forslund and Freidberg [7] predict reflection of most of the ions. A weak shock of the type we consider reflects only a small fraction of the ions, something consistent with the shape of the spectrum in the experiments where the sharp boundary on the high energy side is consistent with only part of a thermal distribution being reflected. We take an electron temperature of 1 MeV and assume that the already heated and expanding ions are at 2 keV, so that $T = 500$. With a Mach number of 1.35 we obtain a ramp length of about 3 μm if we take $n = 10^{26}$ m^{-3} , while the peak electric field is around 8×10^{11} V/m. To compare with the experimental results, we look at the energy spectrum of the reflected ions. Adding the measured expansion velocity of $0.1c$ to the reflected ion velocity we get the spectrum shown in Figure 2. In both the energy spread of the beam and its detailed shape we obtain a striking correspondence with the experiment. Other recent experiments [9] have shown much broader ion energy spectra, more consistent with the computer simulations and with the behaviour to be expected at higher Mach number. We suggest then that it should be possible to control the ion spectral width if the Mach number of the accelerating shock can be controlled. With low Mach number shocks a narrow spectrum will be obtained by reflecting a small part of the incoming ion distribution though this, of course, will be at the cost of a limited number of reflected ions. To connect with experiments described in literature, we note that the results of Haberberger *et al.* [6] show mono-energetic ions and closely resemble those expected from a laminar shock while those obtained by Najmudin *et al.* [9] show a broader spectrum expected from a turbulent shock structure.

5 Species separation

Finally we look at the possibilty of species separation by such weak shocks if they arise in laser compressed targets. It is easy to add a second ion species tto the theory or, indeed, any number of ion species and carry the theory through in the same way as above. We have looked at the effect of shocks on a plasma which is initially an equal mix of deuterium and tritium at the same temperature. Typical results is shown in Figure 3. Upstream there is a small concentration of reflected ions, mainly the lighter deuterium. Downstream there may be a quite substantial difference in the concentrations, resulting from the greater slowing down of the deuterium. This could have an effect on the efficiency of the fusion reaction.

6 Conclusions

We have developed a theory of low Mach number laminar collisionless shocks in a plasma and have shown that it can give good agreement with observed short length scale electric field structures observed in compressed pellets and with the spectrum of accelerated ions obtained in experiments with multiple pulses incident on a gas target. We have also suggested that if such structures arise spontaneously in compressed ICF targets then the separation of deuterium and tritium they produce may have consequences for the reaction efficiency.

A possible application of laser-driven shock waves like those described in this paper would be in the fast ignition concept. Instead of driving a relativistic electron beam to ignite the pre-compressed fuel a laser beam could be used to drive an electrostatic shock wave that accelerates the ions towards the centre to initiate ignition.

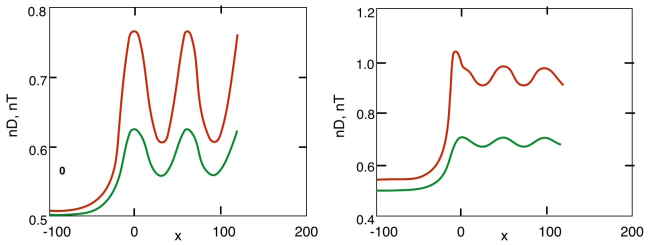

Figure 3: Deuterium (red) and Tritium (green) densities for $T = 20$ and $M = 1.2$ (top) or $M = 1.3$ (bottom).

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