Simulation of photon acceleration technique for density measurement in plasma wakefields

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Introduction

In plasma accelerators, the accelerated beams are pushed forward by the electric field produced in plasma waves (wakefields). The plasma wakefields can produce electric field up to 10-100 GV/m. It is about three orders of magnitude higher than the conventional accelerators can provide [1-4].

One challenge in plasma accelerators development is the inadequacy of plasma wakefield diagnostic techniques. Several methods have been proposed to produce image of plasma density profile. Among them are Frequency Domain Interferometry (FDI) [5], Frequency Domain Holography (FDH) [6,7], and shadowgraphy [8]. These techniques produce qualitative images of plasma wakefield. However, none of them has been used to measure the plasma wakefield amplitude quantitatively.

In this report, we present a simulation result of measurement of plasma wakefield density amplitude using photon acceleration technique [9-11].

Theory

When a photon propagates in a spatially and temporally varying medium, the photon will undergo a change in its frequency and wavenumber. The change in frequency can be related to the plasma density profile. Thus, if the change in frequency is obtained, the plasma density profile can also be obtained.

According to photon ray theory [12], a set of Hamiltonian equations for a photon in a slowly varying medium is as below:

$$\frac{\mathrm{d}\mathbf{r}}{\mathrm{d}t} = \frac{\partial \omega}{\partial \mathbf{k}}, \qquad \frac{\mathrm{d}\mathbf{k}}{\mathrm{d}t} = -\frac{\partial \omega}{\partial \mathbf{r}}, \qquad \frac{\mathrm{d}\omega}{\mathrm{d}t} = \frac{\partial \omega}{\partial t} \qquad (1)$$

where ω and $\mathbf{k}$ denote the frequency and wavenumber of the photon, respectively, $\mathbf{r}$ and t are the position and time in the lab frame, respectively.

For a plasma with natural frequency of ω_p, the dispersion relation for a photon propagating in the plasma is $\omega^2 = \omega_p^2 + |\mathbf{k}|^2 c^2$, where c is the speed of light in vacuum. The plasma frequency in the equation depends on the electron density, n, in the plasma as $\omega_p^2 = ne^2/m\epsilon_0$, where e, m, and ϵ_0 are the universal constants of electron's charge, mass of electron, and permittivity of vacuum space respectively.

In this case, a long laser pulse was sent to propagate together with a previously formed plasma wakefield in z-direction. The perturbed electron plasma density can be represented as a function of laser's wave front position, z, and relative position in laser's frame of reference, $\zeta = z - ct$. Thus, the electron density in the plasma can be written as $n(\zeta, z)$.

Based on the dispersion relation and the third equation of equations **(1)**, we can obtain the frequency change, $\Delta\omega$, of a photon with initial frequency of ω_0 propagating to distance s as [9]:

$$\frac{\Delta\omega}{\omega_0} \approx -\frac{\omega_p^2}{2\omega_0^2}\frac{1}{n_0}\int_0^s \frac{\partial n}{\partial \zeta}\,\mathrm{d}z \qquad (2)$$

where n_0 denotes the initial plasma density without any disturbance. Therefore, we can calculate $\partial n/\partial\zeta$ by obtaining the photon's frequency after it propagates some distance.

Simulation

The simulation was performed using a particle in cell (PIC) code [13] with a simulation software called OSIRIS [14]. The code was run on SCARF-LEXICON machine at STFC Rutherford Appleton Laboratory.

Two laser pulses were sent to the plasma with density of $2 \times 10^{18}\ \mathrm{cm}^{-3}$ in the simulation. The first one was a short pump pulse with duration of 39 fs and normalised intensity of $a_0 = 1.0$ to drive the plasma wakefield. After the short pulse, we also sent a long probe pulse to do the measurement using photon acceleration technique. Duration and normalised intensity of the long pulse were 300 fs and $a_0 = 0.1$, respectively. The wavelengths of both pulses were 800 nm and had propagation distance of 7 mm.

For several propagation distance, we measured the frequency change of the probe pulse and calculated the average density profile of the wakefield, $n_m(\zeta)$. We then compared the measured average density obtained from photon acceleration effect, n_m, and the actual average density profile obtained from the simulation results, $n_a(\zeta)$.

Figure 1 shows the comparison between the measured density and the actual density profile for several propagation distance values. For propagation distance less than 6 mm, the measurement results show well agreements between the measurements and the actual density profiles. However, after propagate more than 6 mm, the measurement results fail to match with the actual values. This is because of photon trapping effect that will be explained in the next paragraph [12].

The photons whose frequency increases will acquire the higher group velocity and vice-versa. This difference in group velocity causes some photons to be trapped at the troughs of the electron density profile and only a small intensity at the peaks. This small intensity at the peaks causes the measurement at these points becomes inaccurate and spoils the measurement. This is the photon trapping effect.

In order to quantify how well the measurement matched the actual values, we calculated the normalised root mean square error (NRMSE) between these values by applying low pass filter to the actual values first to suppress the noise shown in Figure 1(a)-(c). The NRMSE is defined as:

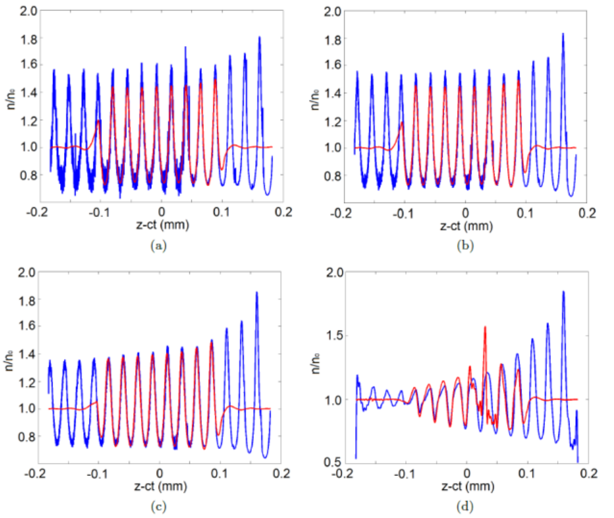

Figure 1. Comparison of measured density (red line) and the actual density averaged over the distance (blue line) when the laser has propagated (a) 0.8 mm, (b) 1.9 mm, (c) 3.8 mm, and (d) 6.3 mm. The measurement only takes place from about –0.1 mm to 0.1 mm relative to the centre of the pulse.

$$NRMSE = \frac{RMSE}{\max(n_a) - \min(n_a)} \qquad \textbf{(3)}$$
$$RMSE = \sqrt{\frac{1}{\zeta_0}\int [n_m(\zeta) - n_a(\zeta)]^2 \, d\zeta}$$

where ζ_0 denotes the range in position where the NRMSE is calculated.

Figure 2 shows the NRMSE values between the measured value, n_m, and the actual value, n_a, at several propagation distance. In the figure, we can see that for propagation distance less than 6 mm, the relative error of the measurement is less than 10%. However, after 6 mm, the relative error goes high and becomes more unstable. This is where the photon trapping effect takes place [12].

Conclusion

We have performed simulation of density measurement of plasma wakefield using photon acceleration. If a photon propagates together with plasma wakefield, the photon will undergo a frequency change. The change in frequency can be measured to obtain to average density profile in plasma. Our simulation results show that the measured average density profile has an error less than 10% relative to the actual density profile obtained from the simulation for some propagation distance before the photon trapping effect gets significant.

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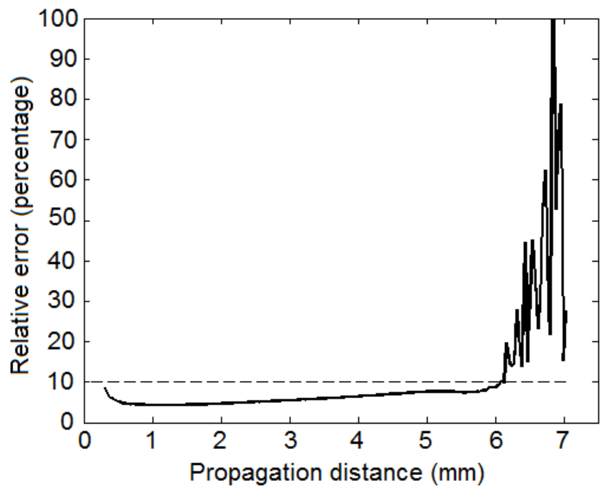

Figure 2. Normalised root mean square error (NRMSE) between measured and actual values from the simulation, shown in percentage. The NRMSE values were calculated from the position –50 μm to 50 μm relative to the probe's centre.

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