# Opportunities for Pair Distribution Function Measurements at XFELs

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## The X-ray Pair Distribution Function

The PDF is a weighted sum of all atom pairs within a material

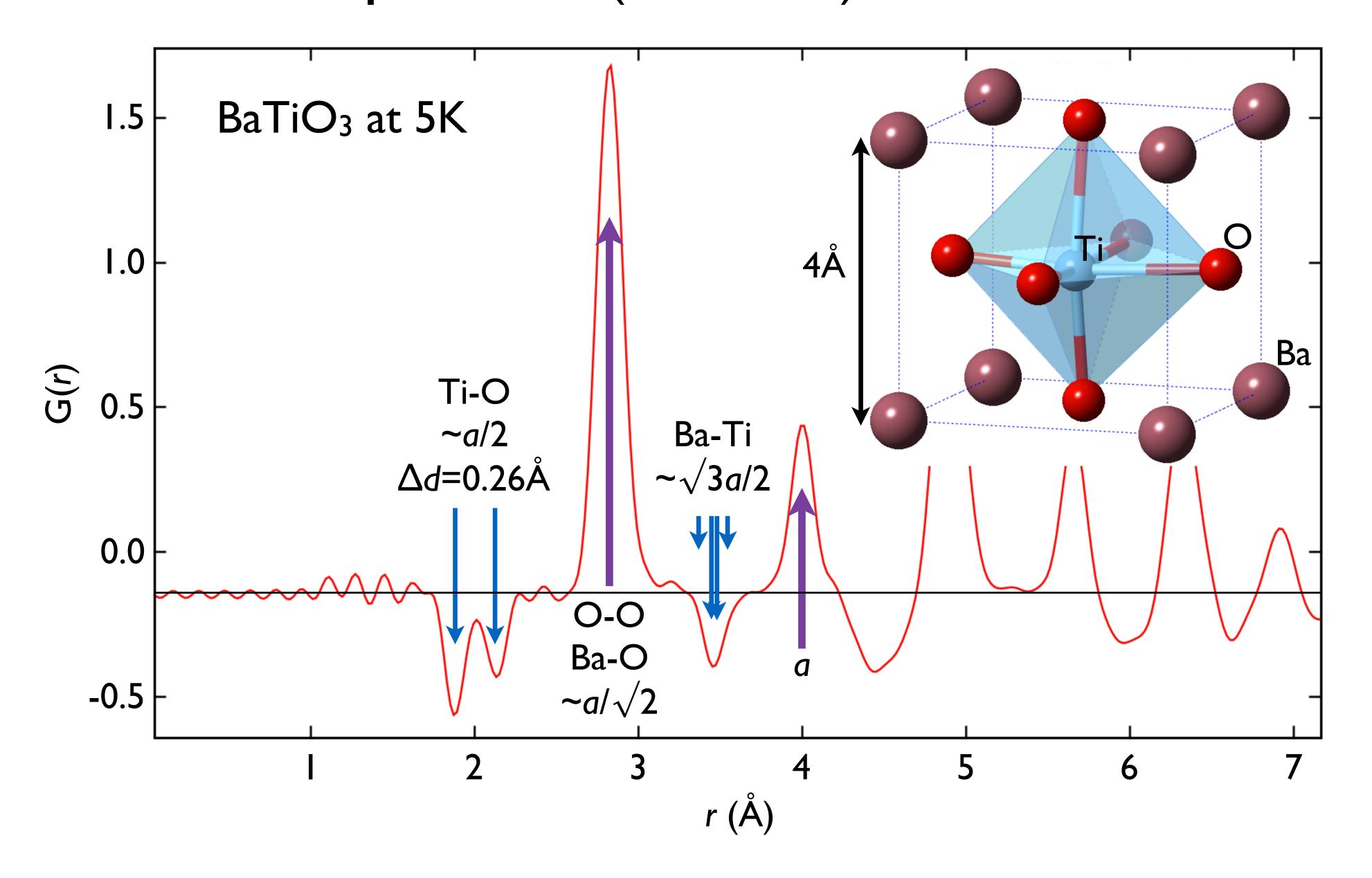
F is a weighted sum of all atom pairs within a material 
$$G^X(r) = \sum_{i,j=1}^n c_i c_j \frac{K_i K_j}{\left(\sum_{i=1}^n c_i K_i\right)^2} [g_{ij}(r)-1]$$
 
$$g_{ij}(r) = \frac{n_{ij}(r)}{4\pi r^2 \, \mathrm{d} r \, \rho_j}$$

Obtained by Fourier transform of the total scattering structure factor

$$G^{X}(r) = \frac{1}{(2\pi)^{3} \rho_{0}} \int_{0}^{\infty} 4\pi Q^{2} F^{X}(Q) \frac{\sin Qr}{Qr} dQ$$

- You can measure it—and calculate it—for any material in any state
- It is especially effective when local structure breaks away from the periodic structural description

### An example: The (neutron) PDF from BaTiO3



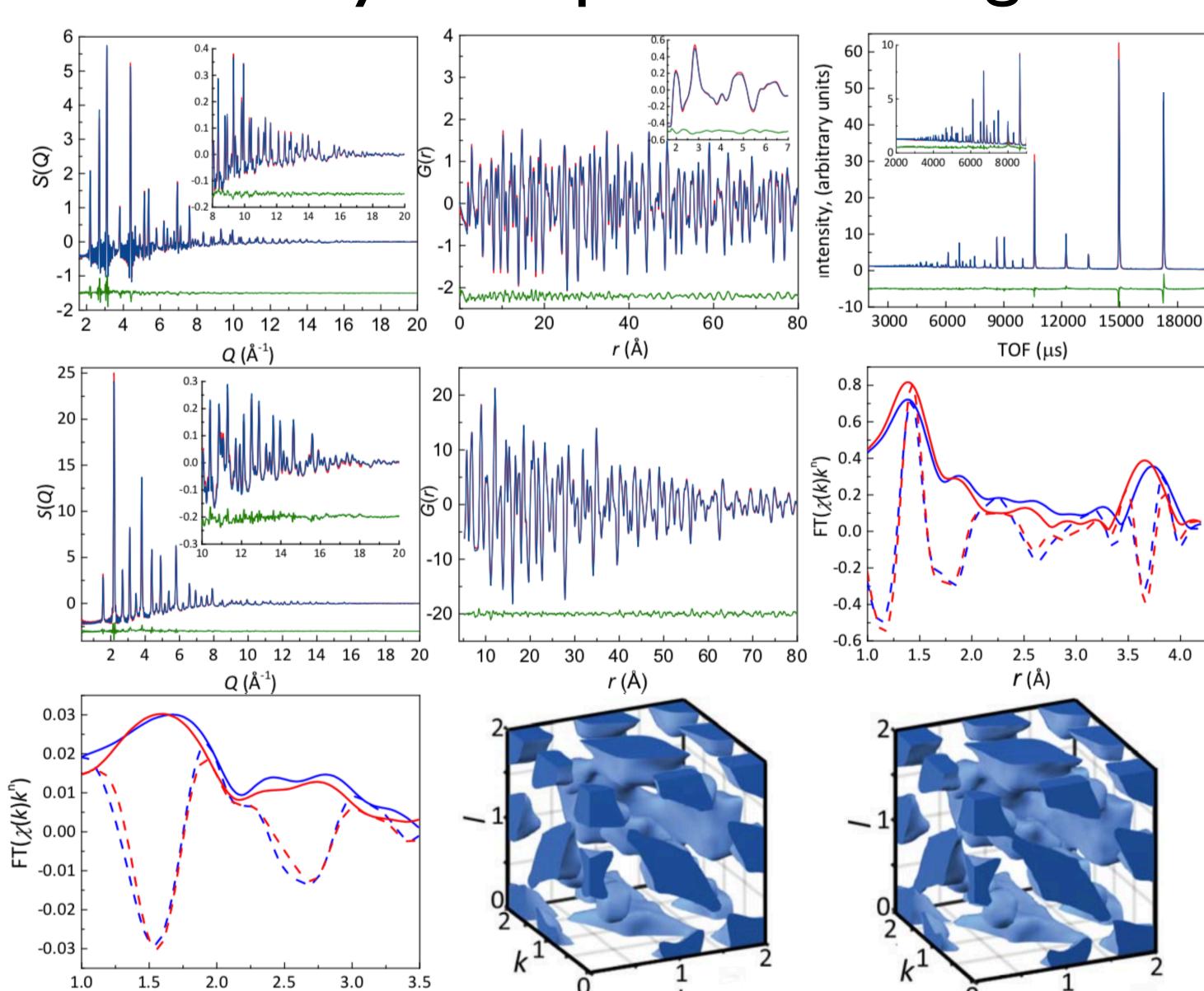
# Fitting PDF etc. in analysis of polar nanoregions in PMN

Neutron
S(Q), PDF
and Bragg profile

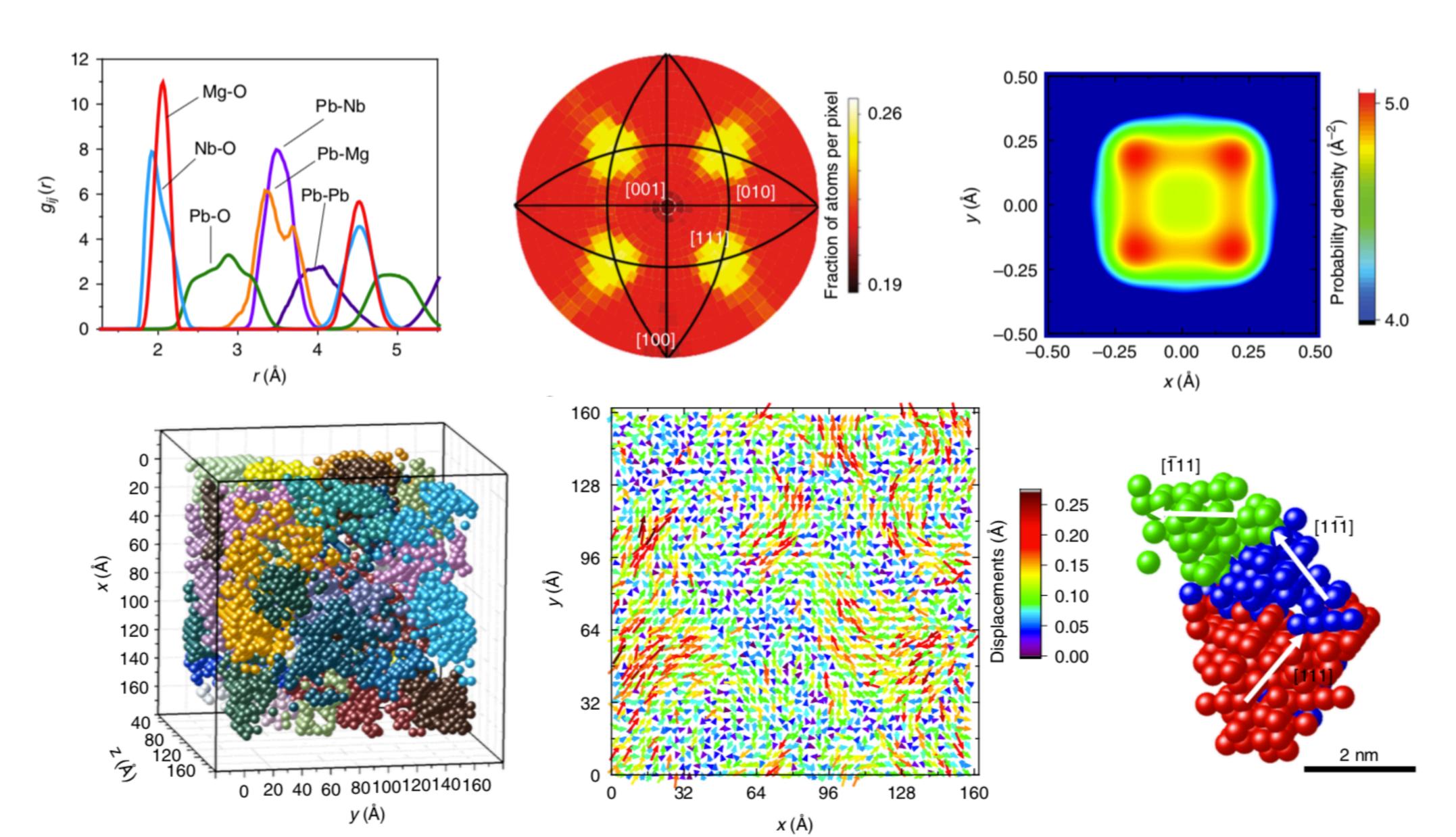
X-ray
S(Q), PDF
and Nb EXAFS

Pb EXAFS; X-ray diffuse scattering

r (Å)



# Polar nanoregions in PbMg<sub>1</sub>Nb<sub>2</sub>O<sub>3</sub> (PMN)



## Other examples?

- Amorphous materials
- Nanoparticles
- Battery materials (ionic conductors)
- Phase transitions
- (Multi-)ferroics
- Negative thermal expansion
- Gas absorption

#### PDF measurements on an XFEL?

Optimising the Fourier transform of the total scattering structure factor

$$G^{X}(r) = \frac{1}{(2\pi)^{3} \rho_{0}} \int_{0}^{\infty} 4\pi Q^{2} F^{X}(Q) \frac{\sin Qr}{Qr} dQ$$

 Where the total scattering structure factor is obtained from the normalised X-ray scattering

$$F^{X}(Q) = \left[\frac{1}{N}\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} - \sum_{i=1}^{n} c_{i} f_{i}(Q)^{2}\right] / \left[\sum_{i=1}^{n} c_{i} f_{i}(Q)\right]^{2}$$

#### PDF measurements on an XFEL?

Optimising the Fourier transform of the total scattering structure factor

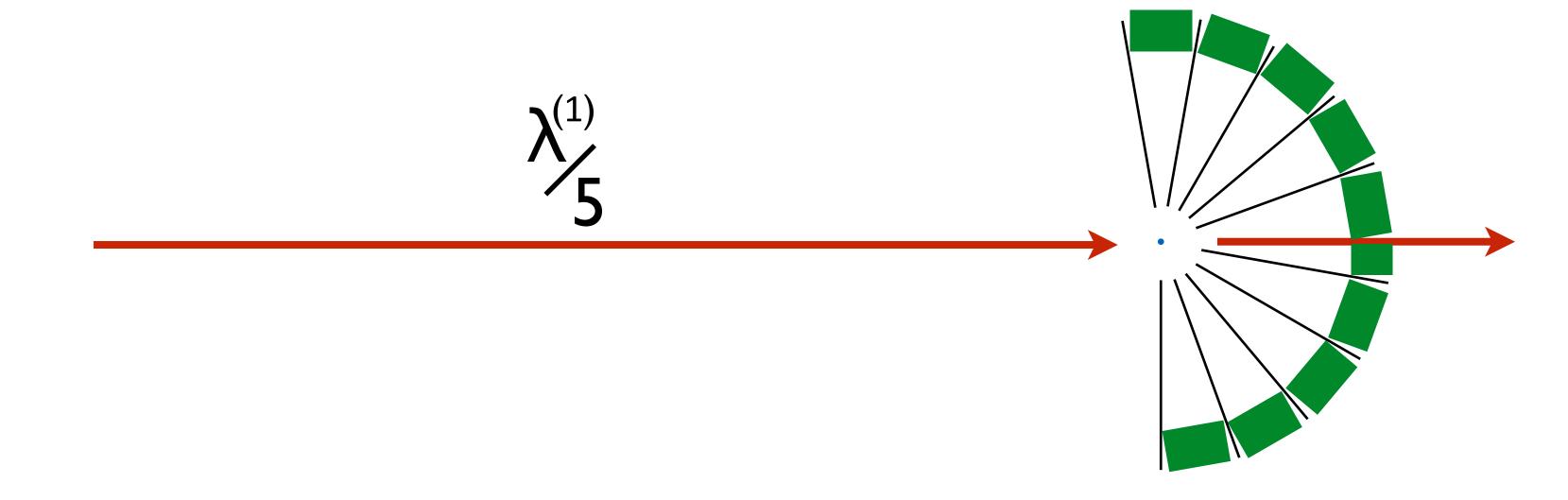
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 Where the total scattering structure factor is obtained from the normalised X-ray scattering

$$F^{X}(Q) = \left[\frac{1}{N} \frac{d\sigma}{d\Omega} \left(-\sum_{i=1}^{n} c_{i} f_{i}(Q)^{2}\right) / \left[\sum_{i=1}^{n} c_{i} f_{i}(Q)\right]^{2}\right]$$

## Requirements for an XFEL PDF instrument...

- Hard X-rays use of higher (3<sup>rd</sup> or even 5<sup>th</sup>) harmonics.
- Wide detector coverage
- Stability

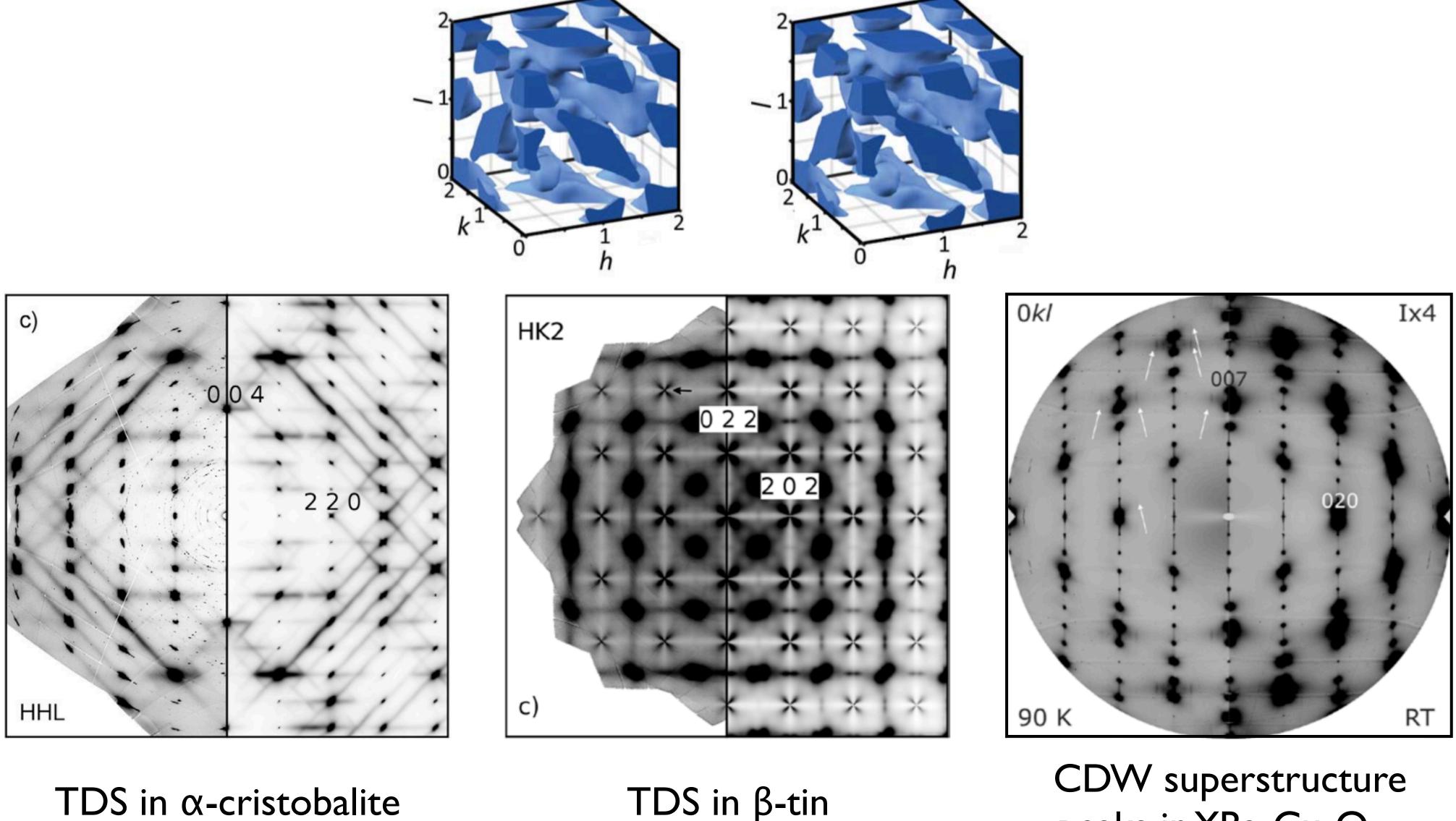


## What PDF experiments might we envisage?

- Amorphous materials
- Nanoparticles
- Battery materials (ionic conductors)
- Phase transitions
- (Multi-)ferroics
- Negative thermal expansion
- Gas absorption

Any experiment where the local structure changes 'quickly' in response to external 'stimuli'

# P.S. Single crystal diffuse scattering?



peaks in YBa<sub>2</sub>Cu<sub>3</sub>O<sub>6.6</sub>