Scaling laws for laser-plasma interaction derived with photon kinetic theory

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Introduction

In this report, we present a derivation of scaling laws for laser-plasma interaction in one-dimensional geometry, using photon kinetic theory. The interest in scaling laws arises from last year's experimental results on mono-energetic electron acceleration with the Astra laser[1]. These results are considered a major breakthrough for laser-plasma accelerated electron bunches in terms of beam quality. Previously, the energy spectra were found to be typically Maxwellian[2], which made laser-plasma based electron sources of limited interest for applications. After the successful demonstration of mono-energetic acceleration, it is timely to address, among various other issues, the scalability to different laser and plasma parameters. The choice of photon kinetic theory[3] for developing the scaling laws is motivated by its simplicity and the useful analogies between laser pulses and electron beams interacting with plasma due to the phase space representation of the electromagnetic field[4]. The results presented in this paper are preliminary in the sense that only the laser pulse evolution is considered, not the electron acceleration. Also, we are planning to extend the model to three-dimensional geometry.

Basic equations

It is well known that the description of laser propagation in underdense plasma can be simplified[5] by exploiting the disparity between the 'fast' timescale of the optical frequency ω_0 and the 'slow' timescale of the plasma response with characteristic plasma frequency $\omega_p \ll \omega_0$. Here, $\omega_p^2 = 4\pi e^2 n_p/m$ and n_p denotes the background plasma density. On the optical timescale, the plasma response is given by the electron quiver motion, a rapid transverse oscillation in the laser electric field. On the plasma timescale, the laser ponderomotive force due to the light pressure can induce charge separation and excite a trailing plasma wave or wakefield. To describe this wakefield, the quasi-static approximation can be used if the laser pulse envelope evolves slowly in the co-moving frame, with a characteristic frequency $\omega_e \ll \omega_p$, which we will define later. Within the quasi-static approximation, plasma quantities depend on time only through the co-moving coordinate $\zeta = (z-v_0t)/c$, where v_0 denotes the linear group velocity $c(1-\omega_p^2/\omega_0^2)^{1/2}$. In this paper, we adopt the widely used relativistic cold electron fluid model[6] for the plasma, which results in a single equation for the dimensionless electrostatic potential $\varphi = e\Phi/mc^2$

$$\frac{\partial^2\varphi}{\partial\zeta^2} = \frac{\omega_p^2}{2}\left[\frac{1+|a|^2}{(1+\varphi)^2} - 1\right], \quad (1)$$

where $a = eA/mc^2$ denotes the dimensionless envelope of the vector potential A that describes the laser pulse. All plasma quantities can be expressed in terms φ of and a, for example the Lorentz factor of plasma electrons

$$\gamma = \frac{1}{2}\left[1+\varphi+\frac{1+|a|^2}{1+\varphi}\right], \quad (2)$$

and the local plasma frequency Ω_p, which includes relativistic and ponderomotive contributions

$$\Omega_p^2 = \frac{\omega_p^2}{1+\varphi}. \quad (3)$$

The laser pulse is modelled with photon kinetic theory[3], i.e. a representation of the electromagnetic field in terms of classical particles (quasiphotons) with time and frequency coordinates, which obey the ray-tracing equations

$$\frac{d\zeta}{dt} = \frac{ck}{\omega} - \frac{v_0}{c}, \quad \frac{d\omega^2}{dt} = -\frac{\partial\Omega_p^2}{\partial\zeta}. \quad (4)$$

Here, ω denotes the quasiphoton frequency, and k its wave number, which is determined from the local dispersion relation

$$\omega^2 = c^2k^2 + \Omega_p^2. \quad (5)$$

The idea behind photon kinetic theory is that a laser pulse can be represented as a distribution $f_p(\zeta,k,t)$ of quasiphotons, with the ζ-dependence describing the temporal structure of the pulse and the k-dependence containing spectral information. For example, a laser pulse with frequency chirp would be modelled as a distribution of quasiphotons with a correlation between ζ and k[7]. The distribution function is used to calculate the vector potential envelope as follows

$$|a|^2 = \int (f_p/\omega)\ dk. \quad (6)$$

The laser pulse evolution follows from a Vlasov equation, i.e. the quasiphoton distribution is transported in phase space along the ray-tracing trajectories given by (4). Formally, the Vlasov equation follows from Maxwell's equation in the limit $\omega \gg \Omega_p$, i.e. for a plasma response that is slow compared to the laser pulse frequency. The advantage of the photon kinetic method is that the laser pulse dynamics can be tracked on the slowest timescale $1/\omega_e$. The disadvantage is that phase information is lost, which excludes a particular class of laser-plasma interactions, e.g. Raman backward scattering.

Scaling laws

Equations (1) and (4) can be used to derive scaling laws in two different limits, namely the linear ($a_0 \ll 1$) and nonlinear ($a_0 \gg 1$) regimes. The scaling laws are found by rewriting the variables in terms of scaling parameters (ω_0, ω_p and a_0) and scaled variables (I, G, s, ψ, τ). In the linear regime, one may assume that $|\varphi| \ll 1$, so the wakefield equation simplifies to

$$\frac{\partial^2\varphi}{\partial\zeta^2} + \omega_p^2\varphi = \omega_p^2\frac{|a|^2}{2}. \quad (7)$$

Clearly, the plasma response scales linearly with the amplitude of $|a|^2$, so that we rewrite this quantity as $|a|^2 = a_0^2 I$. In the same way, we write $\varphi = a_0^2\psi$. From (7) it is also obvious that ζ scales with the plasma period, so we rewrite it as $\zeta = s/\omega_p$. Thus we find

$$\frac{\partial^2\psi}{\partial s^2} + \psi = \frac{I}{2}, \quad (8)$$

which is seen to contain only scaled variables (I, s, ψ). In the linear regime, the scaling for frequency turns out to be $\omega^2 = \omega_0^2(1+a_0G)$. This can be shown by rewriting the equations of motion as

$$a_0\omega_0^2\frac{dG}{dt} = \frac{d\omega^2}{dt} = -\frac{\partial\Omega_p^2}{\partial\zeta} \approx a_0^2\omega_p^3\frac{\partial\psi}{\partial s} \quad (9)$$

and

$$\frac{ds}{dt} \approx \frac{\omega_p^3}{2\omega_0^2}\left(1-\frac{1}{1+a_0^2\psi}\frac{1}{1+a_0 G}\right) \approx a_0\frac{\omega_p^3}{\omega_0^2}\frac{G}{2}. \quad (10)$$

Here we have assumed that both ck/ω and v_0/c are close to 1, and also that $|a_0^2\psi| \ll |a_0G| \ll 1$. In terms of an invariant time variable $\tau = \omega_e t$ with $\omega_e = a_0\omega_p^3/\omega_0^2$, Equations (9)-(10) take on the simple form

$$\frac{ds}{d\tau} = \frac{G}{2}, \quad \frac{dG}{d\tau} = \frac{\partial\psi}{\partial s} \quad (11)$$

which, like (8), is seen to contain only scaled variables. Thus scaling in the linear regime is seen to work as follows: essentially the same physics is found if the laser pulse duration is scaled proportional to the plasma period (i.e. to $n_p^{-1/2}$), and the interaction time proportional to $1/\omega_e$ (i.e. to $n_p^{-3/2}$). The laser pulse duration scales independent of the pulse amplitude, while the interaction time increases with decreasing pulse amplitude as $1/a_0$.

In the nonlinear regime, we approximate the wakefield equation with

$$\frac{\partial^2\varphi}{\partial\zeta^2} = \omega_p^2\frac{|a|^2}{2(1+\varphi)^2}, \quad (12)$$

so that different scalings $\varphi = \psi$, $\zeta = s/(a_0\omega_p)$ are seen to hold for the coordinate and the potential. In terms of scaled variables, the wakefield equation becomes

$$\frac{\partial^2\psi}{\partial s^2} = \frac{I}{2(1+\psi)^2}. \quad (13)$$

The invariant timescale $\tau = \omega_e t$ with $\omega_e = a_0\omega_p^3/\omega_0^2$ is the same as in the linear regime, but a different scaling $\omega^2 = \omega_0^2(1+G)$ holds for the photon frequency, so that

$$\frac{ds}{d\tau} = \frac{1}{2}\left(1-\frac{1}{1+\psi}\frac{1}{1+G}\right), \quad \frac{dG}{d\tau} = -\frac{\partial}{\partial s}\left(\frac{1}{1+\psi}\right) \quad (14)$$

are the equations of motion in scaled variables. In the nonlinear regime, the same physics is found if the laser pulse duration is scaled proportional to the plasma period *and* proportional to $1/a_0$, which corresponds to shorter pulses at higher pulse amplitude (in such a way that the pulse energy scales inversely proportional to the pulse duration).

Simulation results

To check our scaling laws numerically, simulations with a one-dimensional particle-in-cell code have been performed. This recently developed code has a gridless electrostatic field solver and uses a Green's function approach to solve the wave equation. The advantage of this implementation is that the code is less susceptible to numerical noise than more conventional particle-in-cell codes that use finite-difference time domain (FDTD) solvers[8)] for all electromagnetic field components. Especially for the electrostatic field, interpolation from the grid to the particle positions is known to produce numerical heating, which may result in unphysical particle trapping[9)].

The initial shape of the laser pulse is taken to be Gaussian

$$a(\zeta) = a_0\exp(i\omega_0\zeta - \zeta^2/T^2), \quad (15)$$

where T denotes the laser pulse duration. In our simulations, we have chosen short pulses ($\omega_p T \le 2$) that drive high-amplitude wakefields. The evolution of such pulses has been described before[10, 11)], and can be summarized as follows. As the number of photons in the pulse is conserved, the pulse loses energy to the wake by photon redshifting, i.e. a drop in frequency. This photon redshifting is analogous to the energy loss of electrons in a short bunch that drives a wakefield: photons lose energy in regions where the electrostatic field is decelerating. Due to the drop in frequency, the amplitude of the vector potential (6) increases and the wakefield amplitude increases as well - see Equation (1). The feedback from the wakefield to the laser pulse leads to an explosive instability, as the increase of wakefield amplitude speeds up the photon energy loss. As the electrostatic field varies with ζ, the frequency drop is non-uniform over the length of the pulse: this causes an increase of pulse bandwidth, which may lead to steepening of the $|a|^2$-profile and strong pulse compression. Ultimately, as the pulse bandwidth becomes very large, the explosive instability is saturated by group velocity dispersion, which causes an increase of the pulse length and a decrease of pulse amplitude.

For the first set of simulations, we have chosen $a_0 = 1$, $\omega_p T = 2$ and different values (5, 10, 15, 20) of ω_0/ω_p to check the scaling with density in the linear regime. In Figure 1 we show snapshots of $|a|^2$ at $\tau = 4$. Clearly visible are the steepening of the $|a|^2$-profile and strong pulse compression, especially for the higher values of ω_0/ω_p. In Figure 2 we plot the evolution of a_m^2, i.e. the maximum of $|a|^2$, for the different values of ω_0/ω_p. This plot shows increase and subsequent decrease of a_m^2, as expected from the growth and saturation of the explosive instability. For $\tau \le 1$ the evolution of a_m^2 is identical for all values of ω_0/ω_p, indicating that the dynamics is completely scale-invariant. At later times the scale invariance becomes incomplete, as the evolution of a_m^2 is found to be different for the different values of ω_0/ω_p, probably due to breakdown of one or more of the assumptions that underlie Equation (11). However, the maximum of a_m^2 is always found around $\tau = 4$, indicating that the dynamics, although not identical, is *similar* in all cases, with a similarity timescale given by $\omega_e = a_0\omega_p^3/\omega_0^2$.

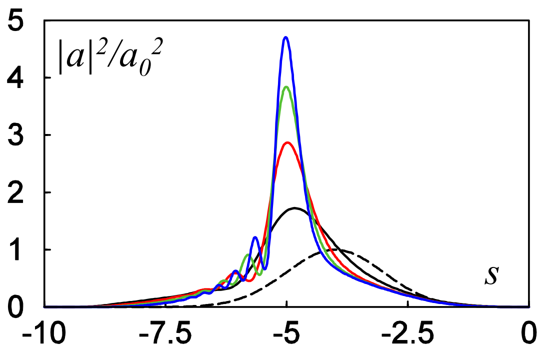

Figure 1. Density scaling in linear regime. Snapshots of $|a|^2$ at $\tau = 4$ for $a_0 = 1$, $\omega_p T = 2$ and $\omega_0/\omega_p = 20$ (blue); 15 (green); 10 (red); 5 (black). For reference, the initial distribution of $|a|^2$ is shown with a dashed black line.

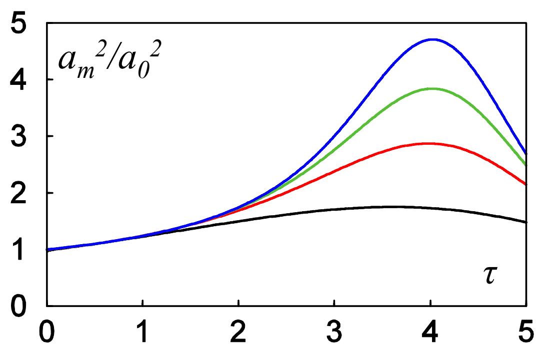

Figure 2. Density scaling in linear regime. Evolution of peak value a_m^2 of $|a|^2$ for $a_0 = 1$, $\omega_p T = 2$ and $\omega_0/\omega_p = 20$ (blue); 15 (green); 10 (red); 5 (black).

The purpose of the second and third set of simulations is to check the scaling with amplitude in the linear and nonlinear regime, respectively. Snapshots of $|a|^2$ are shown in Figures 3 and 5, while Figures 4 and 6 contain the evolution of a_m^2. The results are basically the same as for the first set of simulations, and they confirm the scaling of time with $1/\omega_e$. For small values of the invariant time $\tau = \omega_e t$, a complete scale invariance is found. At larger values of τ, the scale invariance becomes incomplete, and the evolution of scaled quantities is not identical, but similar.

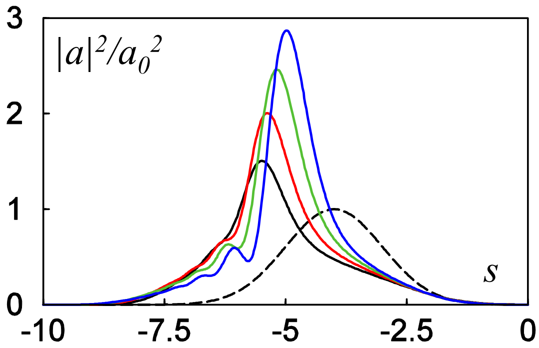

Figure 3. Amplitude scaling in linear regime. Snapshots of $|a|^2$ at $\tau = 4$ for $\omega_0/\omega_p = 10$, $\omega_p T = 2$ and $a_0^2 = 1$ (blue); $a_0^2 = 3/4$ (green); $a_0^2 = 1/2$ (red); $a_0^2 = 1/4$ (black). For reference, the initial distribution of $|a|^2$ is shown with a dashed black line.

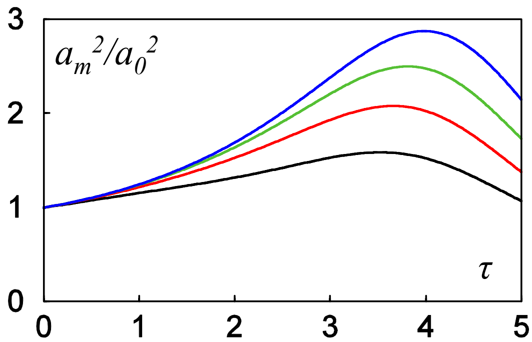

Figure 4. Amplitude scaling in linear regime. Evolution of peak value a_m^2 of $|a|^2$ for $\omega_0/\omega_p = 10$, $\omega_p T = 2$ and $a_0^2 = 1$ (blue); $a_0^2 = 3/4$ (green); $a_0^2 = 1/2$ (red); $a_0^2 = 1/4$ (black).

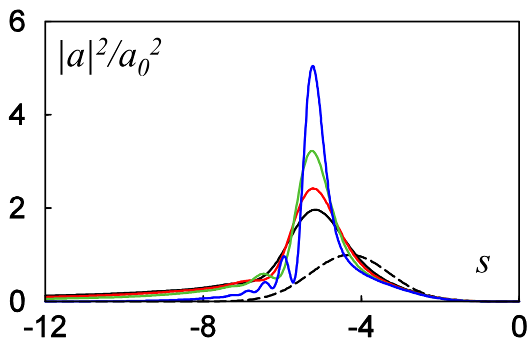

Figure 5. Amplitude scaling in nonlinear regime. Snapshots of $|a|^2$ at $\tau = 3.5$ for $\omega_0/\omega_p = 40$ and $a_0 = 2$, $\omega_p T = 1$ (blue); $a_0 = 4$, $\omega_p T = 1/2$ (green); $a_0 = 6$, $\omega_p T = 1/3$ (red); $a_0 = 8$, $\omega_p T = 1/4$ (black). For reference, the initial distribution of $|a|^2$ is shown with a dashed black line.

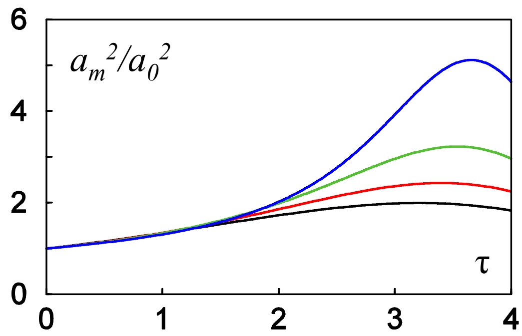

Figure 6. Amplitude scaling in nonlinear regime. Evolution of peak value a_m^2 of $|a|^2$ for $a_0 = 2$, $\omega_p T = 1$ (blue); $a_0 = 4$, $\omega_p T = 1/2$ (green); $a_0 = 6$, $\omega_p T = 1/3$ (red); $a_0 = 8$, $\omega_p T = 1/4$ (black).

Conclusion

In this paper we have studied scale invariance in the interaction of short laser pulses with underdense plasma in one-dimensional geometry, analytically with photon kinetic theory and numerically with particle-in-cell simulations. The simulation results confirm the analytic scaling of the interaction time with laser pulse amplitude and plasma density, namely proportional to $1/\omega_e$ with $\omega_e = a_0\omega_p^3/\omega_0^2$. For small values of $\omega_e t$, the dynamics is found to be completely scale-invariant, while at larger values of $\omega_e t$ the dynamics is still qualitatively comparable. This work is the first application of photon kinetic theory to the derivation of scaling laws: in future work, we plan include three-dimensional effects.

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