Raman amplification of short laser pulses

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Introduction

Stimulated Raman scatter of energy from a long pump pulse into a short seed pulse at a slightly lower frequency has been suggested as a means of producing very intense short pulses in a way which circumvents the need for large and expensive gratings[1-3]. In the linear regime, the Langmuir waves left behind the pulse interact with the pump and the result is a broad pulse, filling the entire region between its starting point and its leading edge. The peak lies approximately in the middle of this region, and so moves at a speed of around $c/2$. However, Malkin *et al.*[2] have shown that there is a nonlinear solution of the appropriate equations representing a self-similar pulse. The amplitude of this is proportional to the distance travelled while the width is inversely proportional to the distance travelled. The energy of the pulse thus increases linearly with distance travelled and energy is fed into it from the pump at a constant rate. The pump is totally depleted by the pulse, so there is no downstream excitation. Recently Ersfeld and Jaroszynski[4] have shown that similar scaling (superradiance) can be obtained in the linear regime if the pump frequency is appropriately chirped. In this case the frequency mismatch downstream inhibits the growth of the pulse. Our purpose here to show that these various regimes can be brought together within a unified description, so that the transition from linear to nonlinear behaviour can be studied as well as the combined effects of nonlinearity and a chirped pulse. We also present some preliminary results showing how the self-similar solution outlined in the first part of this paper is related to more general solutions.

The self-similar solution

Raman scattering is a three wave interaction process in which the highest frequency wave, the pump, gives energy to a lower frequency pulse and a Langmuir wave. We treat a simple one dimensional system for which the equations describing the evolution of the pulse, the pump and the Langmuir wave amplitudes can be written in the familiar form

$$\begin{aligned}
&\frac{\partial a}{\partial t}+\frac{\partial a}{\partial z}=bn\\
&\frac{\partial n}{\partial t}-2i\beta zn=b^{*}a\\
&\frac{\partial b}{\partial t}-\frac{\partial b}{\partial z}=-n^{*}a
\end{aligned}\tag{1}$$

with a, b, n the pulse, pump and Langmuir wave amplitudes respectively. The scaling is such that frequency times amplitude squared is proportional to the wave energy density, time is in units of the inverse linear growth rate based on the initial pump amplitude and velocity is scaled to c. We assume a very underdense plasma in which the group velocity of both transverse waves can be taken as c. In order to introduce frequency chirp into the pump, we follow Ersfeld and Jaroszynski[4] and assume that

$$b(z,t)=B(z,t)\exp\left(\frac{i\beta}{2}(z+t)^{2}\right)\tag{2}$$

with $B(z,t)$ a slowly varying function of its arguments. Guided by the linear theory we make the changes of variable

$$A=a\exp\left(-\frac{i\beta}{2}(z-t)^{2}\right)\qquad N=n\exp(2i\beta zt)$$

to obtain

$$\begin{aligned}
&\frac{\partial A}{\partial t}+\frac{\partial A}{\partial z}=BN\\
&\frac{\partial N}{\partial t}-2i\beta zN=B^{*}A\\
&\frac{\partial B}{\partial t}-\frac{\partial B}{\partial z}=-N^{*}A
\end{aligned}\tag{3}$$

and look for a solution of the form

$$\begin{aligned}
&N(z,t)=F(\varsigma)\\
&A(z,t)=zG(\varsigma)\\
&B(z,t)=H(\varsigma)
\end{aligned}\tag{4}$$

with $\zeta = z(t-z)$. The set of equations (3) then becomes

$$\begin{aligned}
&\varsigma G'+G=HF\\
&F'-2i\beta F=H^{*}G\\
&(3z-t)H'=-zF^{*}G
\end{aligned}\tag{5}$$

The first two of these give equations in the variable ζ, but the third is not in this form. Now, however we can follow the procedure of Malkin *et al.*[2] and note that if the pulse is short then H' is only non-zero when $z \approx t$, so that we can approximate our equations by

$$\begin{aligned}
&\varsigma G'+G=HF\\
&F'-2i\beta F=H^{*}G\\
&2H'=-F^{*}G
\end{aligned}\tag{6}$$

a set of ordinary differential equations in ζ. This solution is, of course, a rather special solution of the set of equations (3), which will have a wide variety of solutions for different initial conditions. The question of what initial condition produces this solution can be answered using Laplace transform techniques to analyse the linear regime. We put $B = 1$ and take the Laplace transform with respect to time of the first two equations in (3). The initial condition is be taken to be

$$\begin{aligned} A(z,t) &= \delta(z) \\ N(z,0) &= 0 \end{aligned}, \qquad (7)$$

corresponding to a very short initial seed pulse and a quiescent plasma with no Langmuir wave present. The solution for $z \geq 0$ is then found to be

$$A(z,t) = \frac{z}{2\pi i}\int_C e^{i\varsigma}\left(1 - \frac{2i\beta}{q}\right)^{-\frac{1}{2i\beta}} dq \qquad (8)$$

where C is the usual Bromwich contour, parallel to the imaginary axis and to the left of all singularities of the integrand. As it stands, this integral is divergent, the reason being that it contains the initial δ-function. If the integral representation of this is subtracted, then we obtain a convergent integral. Equation (8) and the associated solution for N are of the form given in (4), so we conclude that the similarity solution is generated by the initial conditions (7).

The only parameter in (6) is the chirp rate (normalised to the square of the linear growth rate in the scaling we use). If it is put equal to zero we get the solution of Malkin *et al.*, except that their similarity variable is $t(z - t)$. This makes little difference when the pulse is localised around $z = t$, but our solution connects smoothly to an exact solution in the linear regime, when the pulse initially broadens. It also goes into the linear solution with a chirp, and so brings the linear and nonlinear regimes, with or without a chirp, into a unified theory.

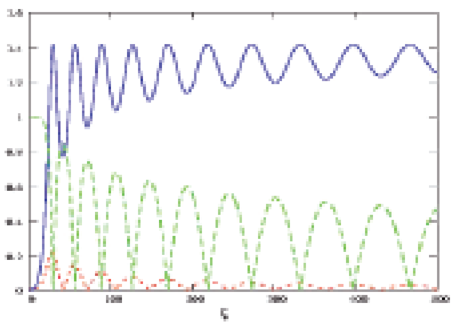

Figure 1. The amplitudes of F, G, H (blue, red, green respectively) with no chirp. Asymptotically, G and H (giving the pulse and pump amplitudes) tend to zero, while F tends to $\sqrt{2}$.

The solution of the set of equations (6) with no chirp is illustrated in Fig. 1. In order to get a non-zero solution we need to take a non-zero initial value of the pulse. However, since there is rapid exponential growth initially, the exact value chosen does not strongly affect the solution. After a time t, the region of ζ which is of interest is in the range $[0, t^2/4]$. For small times this lies within the initial region of growth and we obtain a smooth pulse whose maximum lies close to $z = t/2$. At later times we enter the region with oscillations and the behaviour of the pulse is as in the solution of Malkin *et al.*, ie a main pulse whose amplitude increases proportional to time and width varies inversely with time, followed by a train of pulses of decaying amplitude. The effect of a chirp is to produce incomplete depletion of the pulse and a resultant reduction in the pulse amplitude as shown in Fig.2. In the linear regime the solution, as in Fig.3, behaves in a qualitatively similar way if a chirp is present, but unless the chirp is large enough, neglect of the pump depletion leads to pulse and Langmuir wave amplitudes which are too large. Pulse amplitudes corresponding to these solutions are shown in Fig.4.

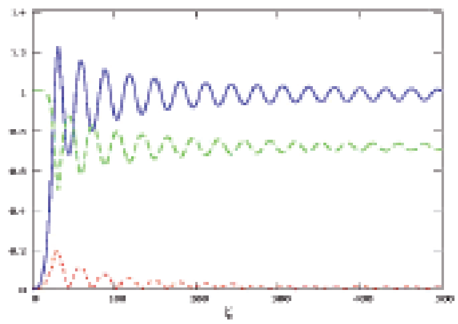

Figure 2. As Fig. 1, but with a chirp of 0.1. Note the incomplete depletion of the pulse.

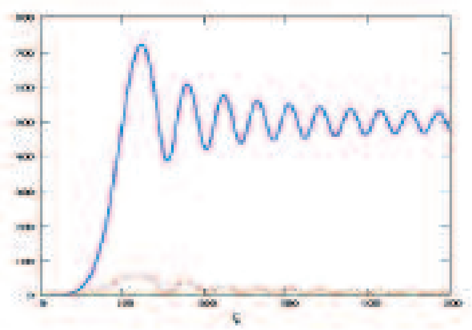

Figure 3. The linear solution with a chirp of 0.1. The value of H is fixed at 1. The behaviour of F and G is qualitatively similar to the nonlinear solutions, but the amplitudes are far too high.

Only if the chirp parameter goes above about 0.2 does the pump depletion become negligible as illustrated in Fig.5. This is, of course, a regime in which there is little transfer of energy from the pump to the pulse. However, a strong chirp may well allow experimental demonstration of the superradiant scaling in circumstances where the available combination of pump intensity and plasma length does not allow access to the nonlinear regime. Also, since chirped pumps are generally what is available, the effect of the chirp on the amplification process is important. As can be seen by comparing the top two plots in Fig.4, an important effect is to reduce the pulse amplitude.

Solution with a more general initial condition

As pointed out above, the similarity solution here corresponds to growth from a small δ function seed. In order to see the effect of a finite width initial seed, we

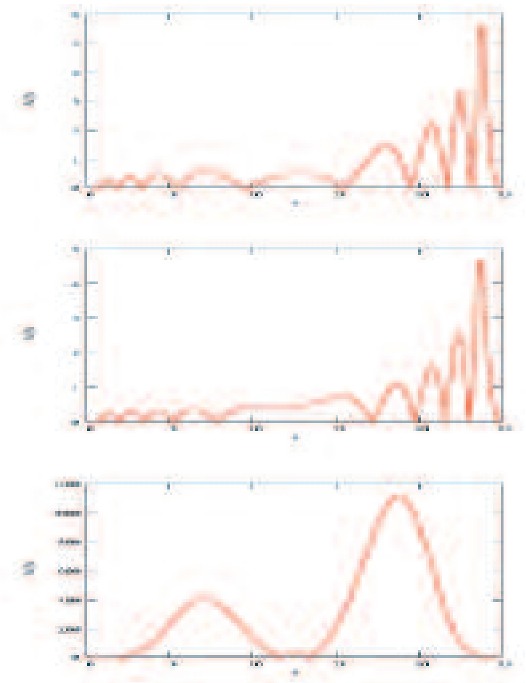

Figure 4. The pulses at *t*=25. Top - nonlinear, no chirp; middle- nonlinear, chirp 0.1; bottom - linear, chirp 0.1.

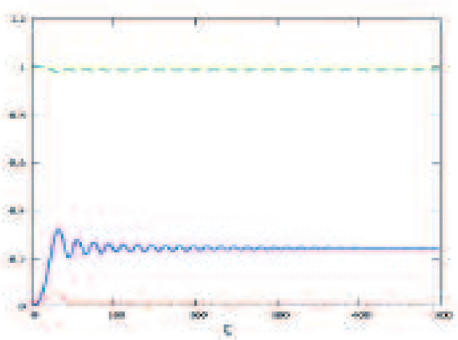

Figure 5. The nonlinear solution with a chirp of 0.2. Now there is little pump depletion and the solution is very close to the linear solution.

return to the initial equations and change the spatial variable to $\xi = z - t$, so that we are in a frame moving with the pulse. So far we have looked at this problem in the absence of a chirp, in which case we obtain

$$\frac{\partial a}{\partial t} = bn \qquad \frac{\partial n}{\partial t} - \frac{\partial n}{\partial \xi} = b^* a \qquad \frac{\partial b}{\partial t} - 2\frac{\partial b}{\partial \xi} = -n^* a \tag{9}$$

Then, essentially following the procedure of Malkin *et al.*, we assume that the Langmuir wave and the pump do not change significantly in the time it takes the pulse to pass a given point, allowing us to drop the time derivative in the last two equations. We can then assume all the variables to be real and take

$$b = \cos\left(\frac{u}{2}\right) \qquad n = \sqrt{2}\,\sin\left(\frac{u}{2}\right) \tag{10}$$

with $u = u(x,t)$ obeying

$$\frac{\partial u}{\partial \xi} = -\ 2a(\xi,t) \tag{11}$$

The appropriate initial condition is that u=0 upstream, so we have

$$u(\xi,t) = \sqrt{2}\int_{\xi}^{\infty} a(\xi',t)\,d\xi' \tag{12}$$

and also

$$\frac{\partial a}{\partial t} = \frac{1}{\sqrt{2}}\sin u \tag{13}$$

For a given initial a we can use (12) to find u (infinity for the present purposes being the upstream edge of the pulse), then (13) to update the value of a to the next time step. This gives a very simple and quick algorithm to determine the evolution of the pulse.

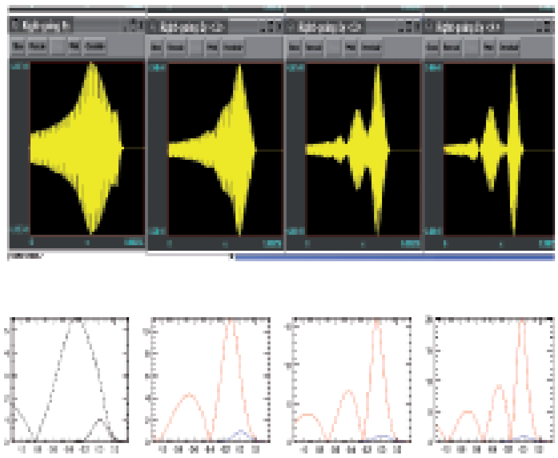

Figure 6. Evolution of a pulse. The top row shows the time evolution from a one-dimensional particle-in-cell code, while the bottom row shows the corresponding results for the pulse amplitude from the analytic approximation. The initial pulse is shown in the bottom set of graphs.

In Fig. 6 we show typical behaviour, comparing the analytic results with one-dimensional PiC simulations of the same system. Clearly the qualitative behaviour is similar and if the normalised units of the analytic results are translated into the units of the simulation there is also reasonable agreement in the pulse width and amplitude. It is evident that the behaviour seems to be evolving towards the self-similar solution described in the first part of this paper. That this is to be expected can be seen by noting that the pulse amplitude at any point can only depend on the initial value of the pulse amplitude upstream of this point (in a frame moving with the pulse). As the seed pulse evolves the most important feature becomes the narrow leading pulse in a train of pulses of decaying amplitude. As this leading pulse becomes shorter, it depends on an increasingly narrow region at the leading edge of the initial pulse and so the solution with an initial δ-function would be expected to become a good description of it.

Conclusions

We have brought together previous similarity solutions predicting superradiant scaling of a short pulse amplified by Raman scattering via a long counter-propagating pump. A single set of equations can describe the evolution of the system in the linear and nonlinear regimes with or without a frequency chirp on the pulse. This brings together regimes treated separately in previous work and lets us see how as the chirp rate increases we go from a system dominated by nonlinearity into a linear regime dominated by the chirp. We have also presented both approximate analytical and simulation results to suggest that any initial pulse will eventually evolve so as to behave like the similarity solution, at least at its leading edge.

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