Energy transfer rates in dense two-temperature plasmas with degenerate electrons

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Introduction

Laser and ion beams are an often used technique to create dense, strongly coupled plasmas in the laboratory. However, these drivers do not produce equilibrium systems, but deposit their energy mostly into the electrons. On the other hand, shocks rather heat the ionic subsystem than the electrons. In both cases, the result is a two-temperature electron-ion system with (in many cases) degenerate electrons and warm, but strongly coupled ions. The subsequent relaxation towards equilibrium reveals many insights into the elementary processes in the warm dense matter regime and is therefore studied intensively using both theoretical [1-7] and experimental [8-10] methods.

The theoretical description of the relaxation process in dense, degenerate plasmas is particularly challenging since it requires the simultaneous consideration of quantum degeneracy for the electrons and strong coupling effects for the ions. Both effects modify the scattering processes and the collective modes in the system. Moreover, the relaxation drives changes in the plasma composition or the correlation energy; in turn, the thermal energy is again modified by the time-dependent binding [11-13] or correlation energies [14].

In this contribution, we focus on the energy transfer between electron and ion subsystems. During the relaxation the electrons must be described as a partially to highly degenerate Fermi fluid for the high densities and moderate heating considered. Due to the blocking of occupied states, one expects a strongly reduced electron-ion collision frequency compared to the classical picture. However, the shortcomings of the classical description, as the often used *ad hoc* cut offs in the impact parameter, result in a break-down of the description at even lower densities. Accordingly, the predicted rates strongly depend on the cut offs demanding a more rigorous theoretical approach.

Theoretical Description

The first description of the energy transfer rates in two-temperature plasmas was given in the seminal works of Landau and Spitzer [1,2]. Due to the use of classical Boltzmann statistics, this approach can however not be applied for plasmas with degenerate electrons. By considering Fermi distributions for the electrons, a first step toward the description of degenerate systems has been done by Brysk [3]. The temperature equilibration is then described by

$$\frac{dT_e}{dt} = \frac{8m^2 Z^2 e^4 \ln\lambda_C}{3\pi \; m_i h^3}\left[\exp(-\mu / k_B T) + 1\right]^{-1} \qquad (1)$$

The occurrence of the Coulomb logarithm $\ln \lambda_C$ is a relict of the classical treatment of collisions still used by Brysk and occurs since Pauli blocking during the collisions is not considered in formula (1). Since the Coulomb logarithm is the ratio of the upper versus lower cut offs, such a treatment raises the question of how these cut offs should be determined in a strongly coupled system with degenerate electrons. The usual definition as the ratio of the Debye screening length and the deBroglie wave length fails since the latter becomes larger than the first. Therefore, the Coulomb logarithm is often clamped at a given value (e.g. $\ln \lambda_C=3$).

An approach that naturally includes degeneracy effects was introduced by Dharma-wardana & Perrot [5] while the influence of collective modes on the energy transfer was studied. If the coupling between the electrons and ions is fully taken into account, the energy transfer rate is given by

$$\frac{\partial E}{\partial t} = 4\int_0^\infty \frac{d\omega}{2\pi}\omega \int \frac{dq^3}{(2\pi)^3} \left|U_{ei}(q)\right|^2 \frac{\Delta N_{ei}(\omega)\chi_i''(q,\omega)\chi_e''(q,\omega)}{\left|1-\left|V_{ei}(q)\right|^2 \chi_i(q,\omega)\chi_e(q,\omega)\right|^2} \qquad (2)$$

Here, the electron and ion response functions χ_e and χ_i, respectively, are the main input quantities. Furthermore, the bare electron-ion potential is replaced by a pseudo-potential to account for strong scattering and structural arrangements in the plasmas.

The sharp peaks in the response functions χ make a direct evaluation of Eq. (2) practically impossible since the peaks in the numerator and denominator cancel only partially. This behaviour gives rise to the occurrence of a new coupled mode structure. Therefore, the energy transfer is often considered within Fermi's Golden Rule where the denominator is set to unity.

Still, sharp peaks in the integral must be treated in a different way: Following the ideas of Hazak *et al.* [6], one can use the electron response in the low frequency limit since ionic response function rapidly decays. Furthermore, the Bose function describing the mode occupation can be expanded. The remaining ω-integral can then be evaluated analytically by the f-sum rule. Finally, one arrives at [15]

$$\frac{\partial E}{\partial t} = k_B(T_e - T_i)\frac{8n_i Z_i^2 e^4}{m_i}\int_0^\infty dq \left.\frac{\partial \chi_e(q,\omega)}{\partial \omega}\right|_{\omega=0} \qquad (3)$$

This formula can now be evaluated easily.

In the Random Phase Approximation (RPA), the coupled mode formula (2) can also be rewritten in a form that allows a numerical evaluation. To that goal, the response functions were cast in dielectric functions. It can then be shown that the remaining integral still contains sharp peaks at small wave numbers q, however, these peaks are limited in height [16]. Therefore, the weight of the peaks vanishes with decreasing width.

Results and Discussion

Fig. 1 shows the electron-ion energy transfer rate as a function of the electron temperature. Only at high temperatures the different theories agree. Clearly, non of the approaches using Coulomb logarithms is able to describe the energy transfer in the (degenerate) low temperature region: the usual definition of λ_C results in much to small rates due to questionable definition of the cut offs. On the other hand, a clamping of the Coulomb logarithm at a temperature-independent value gives an unphysical increase of the rate (mainly due to the prefactor T_e- T_i). Comparing the the two FGR lines, one observes that the calculation for a degenerate system gives actually higher energy transfer rates than the non-degenerate approach. As reason for this behaviour the different form of the electron distribution can be named the main effect; Pauli-blocking of occupied states is of minor importance.

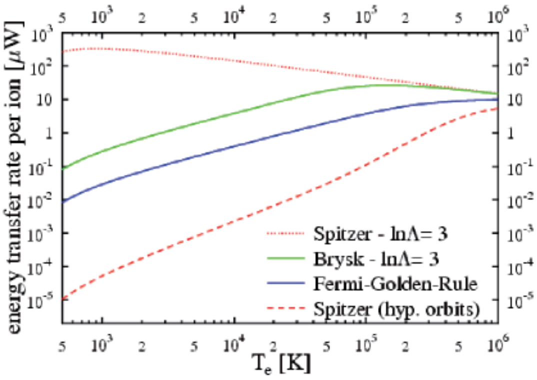

Figure 1. Electron-ion energy transfer rates for a shock-produced silicon plasma with an ion charge of Z_i = 4, an ion density of $n_i = 1.2{\cdot}10^{23}$ cm^{-3} and an ion temperature of $T_i = 10^3$ K.

Up to now, only degeneracy effects were considered and coupled mode effects were left out. The combination of both is considered in Fig. 2. Here, the energy transfer rates in a two-temperature hydrogen plasma are shown for a wide range of electron temperatures. While the Landau-Spitzer and FGR rates coincide for high electron temperatures, the coupled mode formula (2) still gives lower values. This is particularly surprising for the very weakly coupled plasmas considered in the keV range where the Landau-Spitzer approach is believed to be accurate. The difference between the theoretical models strongly increases for lower electron temperatures where the plasma becomes more strongly coupled. They are even larger in the degenerate regime where $T_e < T_i$. Here, the coupled modes results in an energy transfer that is only half the one predicted by the FGR. As expected, the Landau-Spitzer approach fails to produce reasonable results for these parameters.

Summary

We have demonstrated how degeneracy affects the electron-ion energy transfer in dense two-temperature plasmas. Furthermore, coupled modes effects are shown to strongly reduce the temperature equilibration rates.

Acknowledgements

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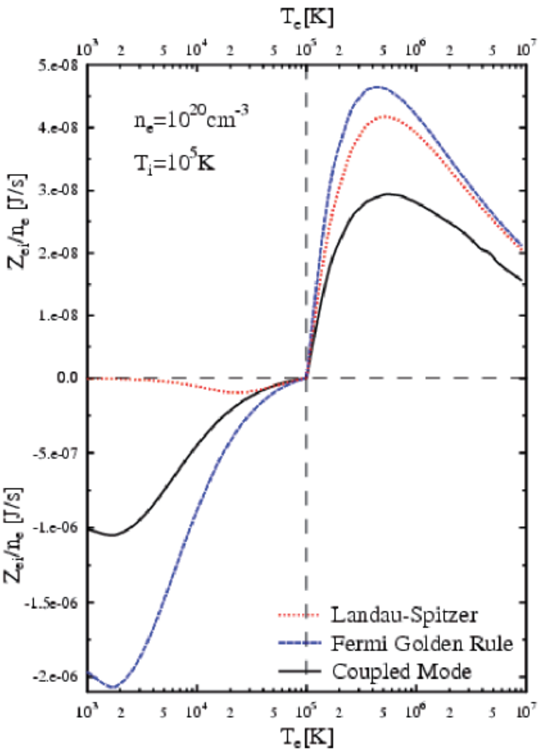

Figure 2. Electron-ion energy transfer rates for a dense hydrogen plasma with a density of n = 10^{20} cm^{-3}.

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