Coherent X-rays from relativistic plasmas

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Introduction

As Nicolaas Bloembergen pointed out in his Nobel lecture 1981 [1], nonlinear optics had developed into a significant subfield of physics. The availability of tunable dye lasers had made detailed nonlinear spectroscopic studies possible throughout the visible region of the spectrum, from 0.35 to 0.9 μm. Conversely, nonlinear techniques extended the range of tunable coherent radiation. However, Bloembergen also emphasized that the soft X-ray region still presented a challenge [1].

We report here how new theoretical ideas and successful experimental research accomplished at the Rutherford Appleton Laboratory (RAL) have solved the problem stated by Bloembergen and paved the way to a tunable coherent X-ray source that can operate in the soft and hard X-ray regions. Such a source is provided by high harmonic generation from relativistic plasmas.

The first observation of high harmonic generation (HHG) from plasma was accomplished in 1981 [2]. A solid target was irradiated by a CO_2 laser and turned into plasma. This resulted in the observation of a number of harmonics limited by the plasma frequency.

A new range of laser intensities was opened up by the development of the CPA technique. This revived the interest in high harmonic generation from plasma. The ultra-short and relativistically intense pulses provided by CPA turn a solid target almost immediately into overdense plasma which starts performing a complicated motion. The plasma electron fluid experiences huge pressure from the electromagnetic radiation and starts moving, driven by the Lorentz force of the laser radiation and the Coulomb attraction to the ions. As a result, the radiation reflected from the plasma contains a high frequency component. By 2004 a number of experimental and theoretical results on relativistic laser-overdense plasma interaction had been obtained. However, neither a theoretical description of the harmonic spectrum nor an explanation of the physical mechanism of harmonic generation was known.

A qualitatively new approach to ultra-relativistic laser-overdense plasma interaction was given in 2006 by the Theory of Relativistic Spikes [3,4]. This theory gave an analytical description of the spectrum of high harmonics from relativistic plasmas, which was also illustrated by particle-in-cell simulations. The analytical predictions of this new theoretical approach based on ideas of relativistic similarity and universality have recently been confirmed experimentally at RAL [5,6].

This Report explains briefly the basic ideas of the Theory of Relativistic Spikes and presents new results about the physical processes that cause the radiation of high harmonics from relativistic plasmas (the detailed analytical treatment can be found in [7]). The analysis of the physical mechanism underlying HHG establishes the robustness of the high harmonic generation process, which prevails in practical experiments where various imperfections occur. As a result, the HHG from relativistic plasmas can provide a tunable source of coherent X-rays of unprecedentedly short wavelengths and high intensity that can be even used to generate single attosecond and sub-attosecond pulses as was first demonstrated in [4].

Relativistic spikes

Let us consider a laser pulse of frequency ω_0, linearly polarized in the y-direction, that is normally incident onto a slab of overdense plasma with surface in the (y, z)-plane. It is well known that the transverse momentum of an electron inside the skin layer is $p_y = eA_y(t, x)/c$, where A_y is the y-component of the vector potential. Consequently, the x-component of electron velocity is

$$v_x = c\,\frac{p_x}{\sqrt{m_e^2 c^2 + p_x^2 + \left(eA_y(t,x)/c^2\right)}} \tag{1}$$

Relativistic similarity theory [7,8] shows that if overdense plasma is irradiated by a laser pulse of ultra-relativistic intensity and dimensionless vector potential a_0 both the transverse momentum of electrons p_y and the momentum p_x perpendicular to the plasma surface scale as $m_e c a_0$. Note that the physical processes causing the HHG always take place by the critical surface where the value of the parameter

$$\frac{\text{local electron density}}{a_0 \times \text{critical density}}$$

is of the order of unity [7].

It is interesting that the momenta p_x and p_y result from apparently quite different physical processes. The momentum p_x is due to the radiation pressure which pushes the electron fluid and forces the Coulomb electron-ion attraction to start restoring the equilibrium. The momentum p_y is due to the surface current that prevents the incident laser pulse penetrating into the plasma. Nevertheless, these two momenta are of the same order of magnitude $m_e c a_0$.

Consequently the velocity of electrons inside the skin layer is about c and their relativistic γ-factor scales as a_0. However, this ultra-relativistic velocity is not directed

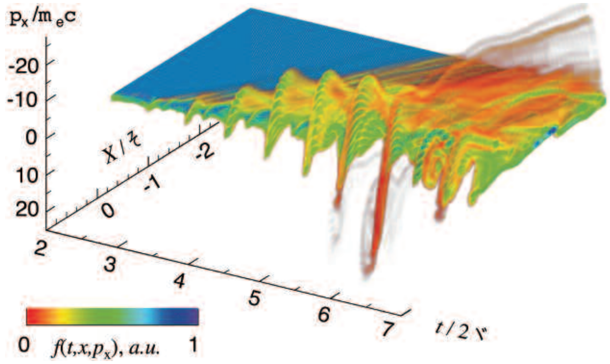

Figure 1. Electron distribution function $f(t,x,p_x)$ as a function of time, x-coordinate in direction of laser pulse propagation and corresponding electron momentum p_x [9]. Electron bursts towards the laser pulse caused by the zeros of the vector potential can be observed clearly. At the same moments of time v_x reaches its maximum, the corresponding γ-factor γ_x jumps up to a_0 and the electrons, located at the zero of the vector potential, emit high frequency photons.

perpendicular to the plasma surface. As a result, the value of the γ-factor corresponding to the electron motion that is normal to the plasma surface

$\gamma_x = 1\Big/\sqrt{1-(v_x/c^2)}$ is

$$\gamma_x = \sqrt{\frac{m_e^2c^2 + p_x^2 + \left(eA_y(t,x)/c^2\right)}{m_e^2c^2 + \left(eA_y(t,x)/c^2\right)}} \qquad (2)$$

One sees that γ_x is usually not as large as a_0 but is of the order of unity.

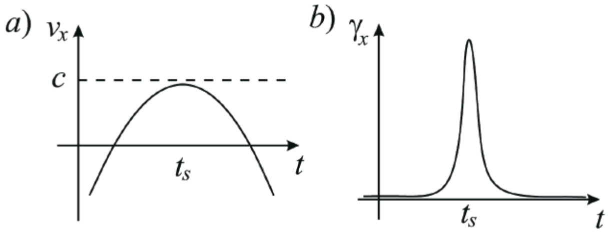

Figure 2. a) The electron velocity v_x is a smooth function reaching the maximum $c(1\text{-}O(a_0^{-2}))$. b) At the moment when vx reaches its maximum the corresponding γ-factor γ_x jumps up to a_0 [3].

However, when the vector potential at the point x passes through zero, the velocities of the electrons at this point and within a small neighbourhood of it are directed perpendicular to the plasma surface (figure 1). Since the vector potential oscillates with frequency ω_0, it passes through zero at x according to the scaling $A_y \sim m_ec^2a_0\omega_0t$. Therefore, the electron velocity v_x at x is a smooth function reaching the maximum value $c(1\text{-}O(a_0^{-2}))$ (figure 2 a).

At the same moment when v_x reaches its maximum the corresponding γ-factor γ_x jumps (these jumps are called ultra-relativistic spikes) up to a_0 (figure 2 b) and the electrons located at x emit high frequency photons. This means that each point of the ultra-relativistic skin layer contributes to the high harmonic generation when the zero of the vector potential passes through it. As a result, the outcoming radiation is accumulated during the whole time the zero of the vector potential travels through the skin layer. To understand this accumulation process one has to notice that the velocity of the zero of the vector potential coincides with the local phase velocity of the signal. On the other hand, the velocity of the vector potential's zero is the velocity of the radiation source. This means that the phase matching condition is automatically satisfied in the whole skin layer and the generated harmonics are in phase [7].

This physical picture has two important consequences. First of all, since the high harmonics are generated by the whole skin layer due to coherent emission of different parts at different times, the skin layer can radiate harmonics of wavelength much shorter than its thickness. Secondly, since the whole relativistic skin layer is involved in this process, the high harmonic generation is robust and is not affected by the surface roughness, as long as this roughness does not destroy the structure of the whole skin layer.

Electromagnetic "shock" waves

The radiation accumulated during the time the zero of the vector potential travels inside the plasma skin layer manifests itself in the form of electromagnetic "shock" waves [7] propagating in vacuum. In these shocks the electric field E_r of the reflected wave depends on time as

$$E_r(t,x) = \text{const}_1 + \text{const}_2 \times (ct-x)^{1/3} \qquad (3)$$

The shock-waves (3) are distinctive in numerical simulations demonstrating the local steepening of the reflected electric field (figure 3). This local steepening

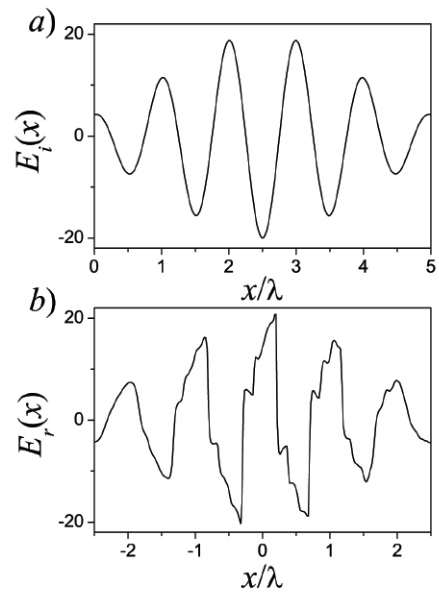

Figure 3. a) Incoming laser radiation. b) Numerical PIC simulation (a_0=20, electron density equal to 90 times the plasma critical density) demonstrates the reflected radiation containing electromagnetic "shocks".

according to the power law $(ct - x)^{1/3}$ is a characteristic feature of the ultra-relativistic γ-spike mechanism.

In contrast to the relativistic Doppler-effect that compresses the pulses and thus causes a frequency upshift, a relativistic γ-spike leads to local steepness without any compression of the incident pulse. It is worth emphasizing that Doppler compression of the pulse and the upshift of all frequencies make the reflected radiation spectrum strongly dependent on that of the incident laser pulse. On the other hand, electromagnetic shock steepening is a local phenomenon resulting in the universality of the harmonic spectrum. The HHG spectrum can be calculated as the Fourier image of the electromagnetic shock:

$$\left|E_r(\omega)\right|^2 \sim \left|\int_{-\infty}^{+\infty}(ct-x)^{1/3}e^{i\omega t}dt\right|^2 \sim \omega^{-8/3} \qquad (4)$$

Universal harmonic spectrum

The universality of the high harmonic spectrum can be understood qualitatively. Let us consider again figure 2. Physically, only the time interval of γ-spiking is responsible for the harmonic generation. Consequently, only the region around the maximal velocity v_x contributes to the harmonic spectrum. As a result, since all smooth functions resemble parabolas around their maxima, the harmonic spectrum is universal: it does not depend on the details of the laser-plasma interaction.

The theory of relativistic spikes allows us to derive a single analytical formula describing the whole universal harmonic spectrum:

$$\left|E_r(\omega)\right|^2 \propto \left(\frac{\omega_0}{\omega}\right)^{8/3} \mathrm{Ai}^2\left(\frac{1}{N}\left(\frac{\omega}{\omega_0}\right)^{2/3}\right) \qquad (5)$$

where Ai is the well-known Airy-function, $N = \alpha^{1/3}n_{cr}$ with $n_{cr}=2/(1-v_s)$ and v_s is the velocity corresponding to the peak of the γ-spike and α is a dimensional parameter characterizing the acceleration of electrons at the time of gamma-spikes. It can be shown [3,7] that N scales as a_0^2 and α does not depend on a_0.

Considering the asymptotic behaviour of Ai(x) one sees that for $\omega<\omega_{roll} = N^{3/2}\omega_0 \sim a_0^3\omega_0$ the spectrum decays according to the power law

$$I(\omega) \propto \omega^{-8/3},$$

which coincides with the Fourier image of the electromagnetic shock (4). This power-law decay continues to frequencies of the order of $a_0^3\omega_0$ and for $\omega>\omega_{roll}$ rolls over into exponential decay [3,7]. The high harmonic spectrum predicted by equation (5) can be beautifully visualized numerically (figure 4).

It is worth emphasizing at this point that the power law decay, the a_0^3-roll-over and the subsequent exponential decay are all consequences of the same physical phenomenon and are described by the same formula (5).

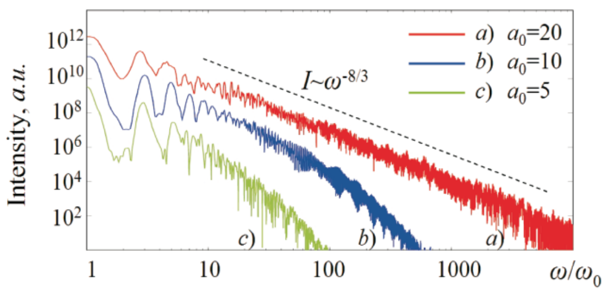

Figure 4. Spectra of the high harmonics generated by laser-overdense plasma interaction. These numerical PIC simulations clearly demonstrate the power-law decay of the harmonic spectrum [3, 7].

Isolated sub-attosecond pulses

The high harmonics from relativistic plasmas present a promising source of ultra-short pulses. Indeed, since all high harmonics are locked in the time domain in the form of an electromagnetic shock, they are highly coherent. In other words their constructive interference results in the generation of extremely short pulses, the duration of which is defined by the harmonic spectrum (4). One readily sees that the pulse duration scales as

$$T \propto \frac{1}{\omega_{roll}} \propto \frac{1}{\omega_0 a_0^3}.$$

Moreover, it is possible to control the development of ultra-short pulses by means of changing the polarization of the incident laser pulse in order to isolate single sub-attosecond pulses [4].

Indeed, the moment of γ-spiking and, consequently, of high harmonic generation, corresponds to a zero of the tangential vector potential A_y. In other words, controlling the polarization of the laser pulse leads to efficient control of the harmonic generation. For linear polarization one

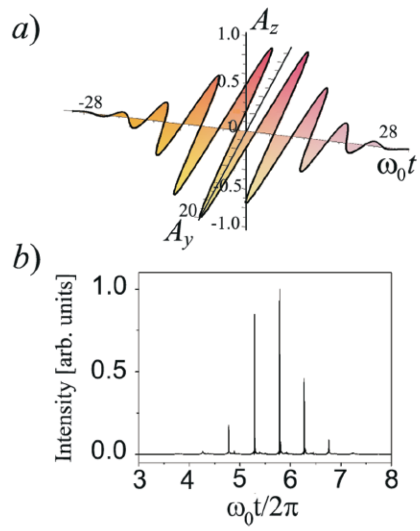

Figure 5. Linearly polarized laser pulse produces a train of ultra-short pulses.

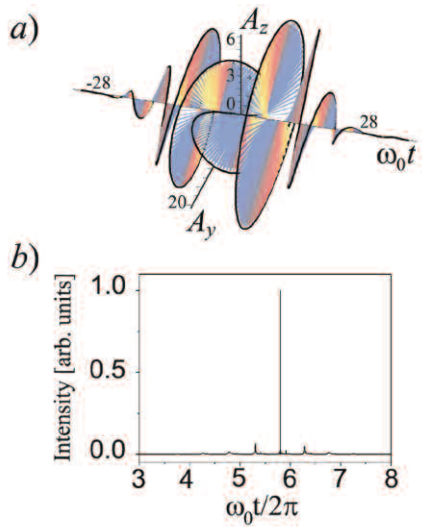

Figure 6. If the polarization of the laser allows only one zero of A_y per laser pulse, one can produce an isolated ultra-short pulse. The tool for manipulating the high harmonic generation from relativistic plasmas using the zeroes of the vector potential is called Relativistic Plasma Control [4].

obtains a train of ultra-short pulses, appearing twice per laser period (figure 5). However, if the polarization of the laser allows only one zero of A_y per laser pulse, one can produce an isolated ultra-short pulse (figure 6). This tool for control of the laser-driven plasma dynamics was presented for the first time in [4] and named Relativistic Plasma Control.

Conclusions

The Relativistic Spikes theory of high harmonic generation at the boundary of relativistic plasmas has demonstrated that the whole skin layer is in a coherent state, thereby enabling it to generate high harmonics with wavelengths much shorter than both the size of the surface roughness and the skin layer thickness. A mechanism to control the generation of high harmonics and produce single ultra-short pulses has been proposed. Consequently, relativistic spikes bringing about high harmonic generation from plasma surfaces form a promising physical phenomenon that can be used in order to develop X-ray time-resolved metrology.

Acknowledgements

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