Scaling relations for recombination following tunnelling ionisation

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Introduction

It has been known for many years that recombination has the potential to induce population inversion and consequently gain. Further if an inversion can be induced with the ground state of hydrogenic ions as the lower level, the resulting transition will be at short wavelength. This objective, however, is hampered by severe difficulties in its practical realization. The recombining electrons must be cold despite the high level of ionisation, and the ground state must be initially totally empty. Tunnelling or above threshold ionisation (ATI) using (preferentially) plane polarized light offers such a potential route forward.

In recent papers [1,2] we investigated both the establishment of a recombination cascade in hydrogenic (lithium) ions by cold electrons, and the ability of ATI to generate such a medium. It was apparent from these analyses, that we may separate the overall process into two distinct steps: first ionisation and establishment of the cold electron distribution; second recombination accompanied by relaxation of the electron distribution by collision. In this note we examine the scaling of these processes using carbon ions ionised by a (KrF) laser of 0.25 μm wavelength and pulse length 50ps with an electron density of 2×10^{20} cm^{-3} as exemplar.

One clear result from earlier work determines an essential characteristic of these ideas, namely that the electron distribution following ionisation of the pure element is too hot to allow inversion to form in the recombination cascade. To this end we follow an earlier proposal [3] to use hydrogen as a buffer gas, which will provide the cold electron background by ATI, due to its low ionisation threshold.

Ionisation phase

Since the atomic field scales as Z^3 for hydrogenic ions of charge Z, the required intensity for tunnelling ionisation scales as Z^6. Thus the necessary intensity to fully strip the ground state of the hydrogen ion using plane polarized light is approximately

$$I_0 \approx 1.4 \times 10^{14}\, Z^6 \text{ W cm}^{-2} \qquad [1]$$

Applying the ADK formula [4] the cycle averaged ionisation probability is

$$\overline{p} = 1.05 \times 10^{13}\, Z^2\, \alpha^{-1/2} \text{ sec}^{-1} \qquad [2]$$

independent of wavelength [5]. The factor

$$\alpha = \frac{2}{3} Z^3 \frac{E_{at}}{E_0} \approx 1.25\times10^{8}\, Z^3 I_0^{-1/2} \qquad [3]$$

where E_{at} is the atomic field and E_0 the peak laser field. From equation (1) we obtain $\alpha \approx 10$. The corresponding peak quiver energy at the intensity given in equation (1) is

$$\varepsilon_q \approx 1.87 \times 10^{-13}\, \lambda^2 I_0^{2} \approx 26.1\, \lambda^2 Z^6 \text{ (eV)} \qquad [4]$$

where λ is the pump laser wavelength in μm. This scales strongly with Z. For ionisation near threshold the electron distribution is approximately one dimensional Maxwellian with mean energy ε_q/α. This indicates the need for a high background concentration of hydrogen, which should be ionised close to threshold (α large). Furthermore since re-collision may lead to energies up to a maximum value $\approx 1.59\, \varepsilon_q$, efforts should be made to avoid this taking place. The electron energy distribution following tunnel ionisation is strongly peaked towards zero energy with a high energy tail.

During the ionisation, phase energy is strongly absorbed by inverse bremsstrahlung. In principle the cold electrons will be most strongly heated, however at high intensities, required to strip the ion, absorption is in the strong field regime, where the rate is almost independent of the electron thermal velocity. Thus all electrons are essentially heated in a similar fashion, driving the distribution towards a Maxwellian form [6]. The average inverse bremsstrahlung absorption gain per electron scales as

$$\varepsilon_a \sim \frac{\overline{Z^2}}{\overline{Z}} n_e I^{-1/2} \qquad [5]$$

where $\overline{Z}$ and $\overline{Z^2}$ are the mean ion charge and the mean squared ion charge averaged over the plasma composition respectively.

The electrons are further driven towards to a Maxwellian by electron-electron collisions, but this effect tends to be weak compared to collisional absorption during this phase.

At the conclusion of this phase the electrons have a distribution which takes the form of a Maxwellian with a hot tail. The Maxwellian cold part of the distribution is identified by a 'mode' temperature corresponding to the peak of the distribution [1], which will control the recombination.

Recombination phase

The recombination phase follows the ionisation phase, essentially starting once the laser pulse has finished. Recombination resulting from three body collisions involving a secondary electron preferentially populates the upper states of the hydrogenic ion. A cascade down through the energy levels follows, driven by electron collisions amongst the upper states and supporting an approximate Boltzmann population, until the 'bottleneck' state is reached at which the upward electron collisions cannot balance the downward. The quantum number of this state is approximately

$$n \approx \sqrt{\{I_Z / kT_e\}} \quad [6]$$

where I_Z is the ionisation energy of the hydrogenic ion Z and T_e the electron temperature. In the cascade to states below the 'bottleneck' radiative decay will progressively dominate collisional. Within this cascade recombination phase, transitions across the 'bottleneck' are collisional rather than radiative, and inversions may be established provided the 'bottleneck' does not lie below the upper level. Clearly this provides an upper bound to the temperature at which inversion can be established [2].

In contrast direct radiative recombination preferentially populates the ground state and may prevent inversion. Consequently there is a minimum electron density at which inversion can occur.

It has been shown [2] that cascade recombination in hydrogenic ions is similar if the electron density and temperature are scaled as

$$n_e \sim Z^7 \quad \text{and} \quad T_e \sim Z^2 \quad [7]$$

provided the depression of ionisation due to the ion density is small.

Figure 1 shows the inversion threshold for carbon at which a population inversion is just formed between the first excited (resonance) and the ground states during a cascade, provided the ionisation depression is small. The results are easily related to different ions by the use of the scaling in equation (7).

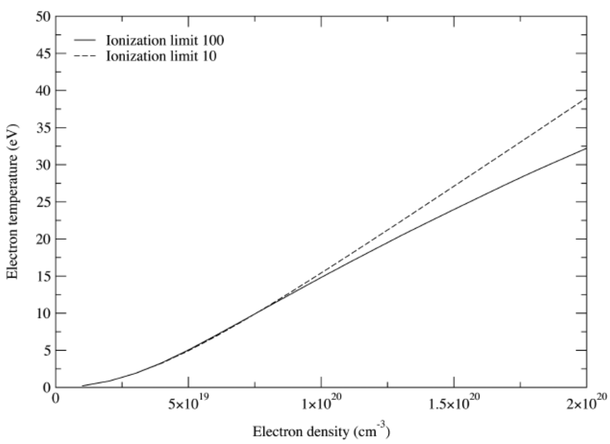

Figure 1. Inversion conditions for carbon with n_{lim}= 100 (effectively no ionisation depression) and n_{lim} = 10.

It is clear from these results that for an inversion to form in carbon the electron density must be greater than about 10^{20} cm^{-3} and the electron temperature less than about 30 eV. This provides quite a stringent limit on the ATI process.

At such high densities we can no longer neglect the depression of the continuum limit, which is given by [7]

$$n_{lim} \approx 1.1 \times 10^4\, Z^{1/2}\, n_i^{-1/6} \quad [8]$$

where n_i(cm^{-3}) is the total ion density. In the case discussed here, the composition is dominated by hydrogen and $n_i \approx n_e$. In carbon at the proposed working density, the limit is given by $n_{lim} \approx 10$. From figure 1, we see that the depression of ionisation slightly eases the limits on inversion. We also note that ionisation suppression does not follow the scaling of equation (7).

During the recombination phase the electron distribution is further modified by electron heating from three body collisions as well as electron relaxation. In the case of carbon we find that both these effects are relatively insignicant.

Application to carbon – ionisation phase

We apply these ideas to the particular case of carbon. To simulate the ATI phase we use the Fokker-Planck model described in ref. [8], which rapidly calculates the mode temperature at the end of the ionisation phase as the composition is changed, but without treating the subsequent recombination in detail. Figure 2 shows the variation of this temperature with electron density as the carbon/hydrogen composition ratio is varied. Also plotted is the inversion condition for ionisation depression of n_{lim}=10 taken from figure 1. It can be seen that inversions can only be achieved if the electron density is sufficiently high, namely greater than 10^{20} cm^{-3}. For a reasonable composition ratio, the density needs to be about 2×10^{20} cm^{-3}.

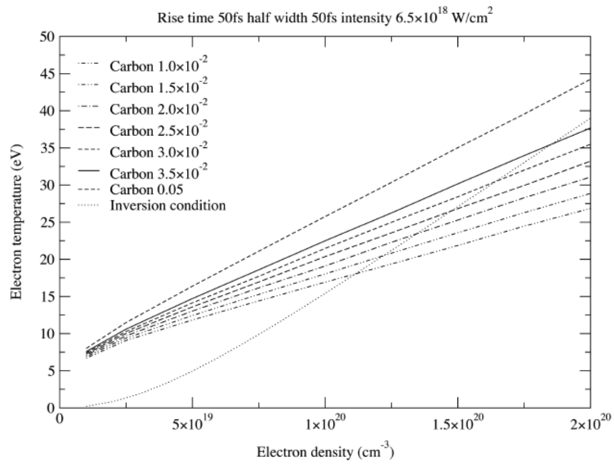

Figure 2. Plots of the mode temperature for varying electron density and hydrogen/carbon composition following ATI by a beam of 0.25 μm wavelength.

The role of inverse bremsstrahlung absorption can be clearly seen in figure 3 where we compare the mode temperature with the temperature resulting from inverse bremsstralung alone. The ATI energy component is obtained by subtracting the collisional absorption from the total. It can be seen that as may be expected, the collisional absorption is dependent on the composition of the mixture due to the $\overline{Z^2}$ term in equation (5), particularly as the density is increased. The ATI component is less affected by the changing composition and nearly independent of density, although at higher density electron relaxation leads to a small increase in the mode temperature. At operating densities the collisional absorption dominates the initial ATI component even for hydrogen alone.

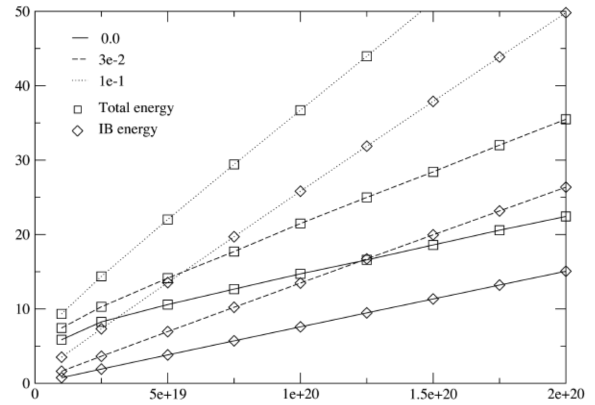

Figure 3. Plots of the energy absorbed during by collisional absorption expressed as a temperature and the mode temperature following ionisation of a carbon/hydrogen mixture of varying composition.

Application to carbon – recombination phase

In order to examine the effects of recombination, the simulation model is modified to include self-consistently the effects of atomic transitions involving three-body and radiative recombination and inter-level collisional and radiative transitions following the prescription in ref. [1]. For the present we consider the effect of an initial Maxwellian distribution of 25eV temperature, i.e. without the hot tail produced by ATI.

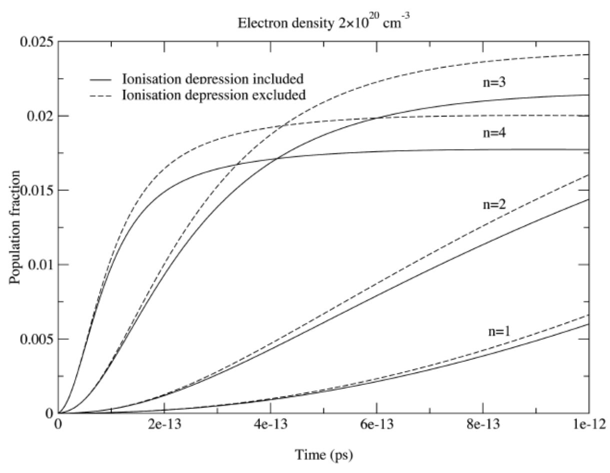

Figure 4. Plots of the population of the levels n=1 to n=4 as functions of time for the cases with and full electron energy relaxation including recombination heating with and without ionisation depression to the level n=10.

From figure 4 we see that the ground state time history is little changed by the inclusion of ionisation depression. This may be anticipated since this population is mainly due to radiative recombination, which is not greatly affected by slight changes in the ionisation energy. On the other hand greater differences are seen as we move to higher quantum levels. This may be understood as the effect of ionisation depression is significantly more marked on the higher levels. Despite the larger reduction in the population of the level n=2 when ionisation depression is included, the inversion is slightly increased. This is due to the differerential effect between the levels n=1 and n=2 resulting from the statistical weight factor.

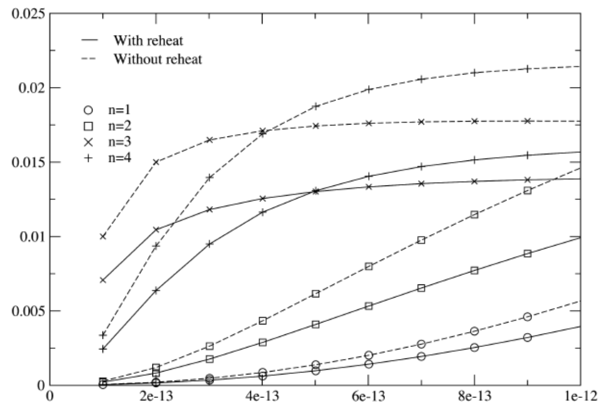

Figure 5. Comparison of the populations generated with and without three body re-heating of the electron distribution.

Although re-heat following three body collisions removes cold electrons, which dominate the cascade, its effect is not large (figure 5). This results from the fact that the level of re-heat which occurs is less than might initially be expected, as collisional effects only give rise to downward transitions amongst the high lying states. Amongst the lower states radiation dominates and the energy is not returned to the electrons, but lost into the external environment. The energy returned to the electron distribution is thus relatively small. As can be seen in figure 5, re-heat decreases the populations of all the states, and leaves the inversion almost unchanged.

Discussion

The parameters, which will allow an experimental demonstration of population inversion and gain by recombination in carbon, are established by the limiting condition for the onset of gain (figure 1). This determines both the electron density and temperature required in the plasma. For (comparatively) high Z ions, the temperature scaling ($\sim Z^2$) is not compatible with the scaling of the electron energy generated by the quiver energies at ionisation for pure materials. In order to achieve sufficiently low temperature, the lasant must be mixed with a high concentration of hydrogen (figure 2), which provides a bath of cold electrons to stimulate the necessary recombination mode.

For the particular case of carbon, which would lase at 33Å, the density must be $\sim 2 \times 10^{20}$ cm^{-3} and the carbon/hydrogen concentration $<3 \times 10^{-2}$. The simulations performed in figure 2 were carried out assuming a pump laser wavelength of 0.25 μm (KrF laser). It is likely that in practice a longer wavelength, e.g. 0.4 µm (2nd harmonic TiS laser) would be used thereby increasing the temperature, and may require a reduced carbon fraction for operation.

The high density necessary for carbon operation presents a further technical challenge as a methane/hydrogen mixture is a likely working medium.

It is essential that the hydrogen be ionised at intensities close to its threshold, equation (1). If this is the case re-collision electrons are weakly generated as the electrons are released only near the peak of the field and will return to the ion with low velocity. Furthermore since the peak quiver energy is low at this stage of ionisation, any re-collision electrons will be relatively cold in comparison with the subsequent inverse bremsstrahlung heating during the higher stages of ionisation. To achieve this it may be desirable to use a separate low intensity pre-pulse to ionise the hydrogen and low stages of the lasant. Unfortunately the Z scaling in equation (2) may require that the hydrogen be ionised with a longer pulse or a pulse, which is slightly excess of threshold if the same pulse duration is used as for the lasant.

We have investigated the roles of electron relaxation and threebody heating in the carbon case, and find that although they make a contribution to the quasi-Maxwellian their role is small. A major determinant is high field inverse bremsstrahlung. Depression of ionisation plays a useful role, changing the populations in the upper state, and thereby relaxing the limiting condition for inversion and increasing the gain.

References

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