Spiral magnetic fields and density cavities generated by hot electron streaming

A. R. Bell

Univsersity of Oxford, Clarendon Laboratory, Parks Road, Oxford OX1 3PU, UK

Contact | **t.bell1@physics.ox.ac.uk**

At very high laser intensities, the energy absorbed by a solid target is given to hot electrons with energies T_h (in eV) in excess of 100keV. The electrons stream into the target with a current density $j_h=I_h/T_h$ where I_h is the energy flux (in SI units) carried by the hot electrons. The current density is in the range 10^{16}-10^{17}Am^{-2}, and this is sufficient to excite plasma instabilities during the laser pulse. Evidence of instability has been seen in foil experiments. The pattern of energy arriving at the rear of the target can be speckled as expected if the hot electron flux breaks up into filaments during propagation (Norreys 2008) [5]. Beam filamentation is also seen in PIC and hybrid simulations, and this is attributed to Weibel-related or resistive instabilities (eg Gremillet *et al.* 2002 [4], Sentoku *et al.* 2003) [6]. Hot electrons stream into the target and thermal electrons stream in the opposite direction providing the neutralising return current. The exponentially growing magnetic field separates the currents into filaments. In its simplest form, WI grows rapidly because the rate of filament separation is governed by electron inertia. However, only a small transverse electron temperature is needed for electrons to traverse the growing filament during a growth time, and WI is then reduced. These effects have been examined by Tzoufras *et al.* (2006) [7], who show that ion motion becomes important and the nature of the instability is changed. WI is just one particular manifestation of a wider range of instabilities and related effects driven by energetic streaming particles (Bret *et al.* 2008) [3].

In the fast ignition (FI) scheme for fusion, the source of energetic electrons creating the fusion hot-spot persists for 10-20psec, which is longer than the sub-psec pulses commonly used in contemporary very high intensity laser-solid experiments. This longer pulse duration provides additional time for ion motion, and instabilities grow over 10 psec which are less active over sub-psec timescales. The hot electrons carrying current j_h are subject to a force of magnitude j_hB where B is the characteristic magnetic field. The thermal plasma, with density ρ, is subject reactively to an opposite force and is accelerated through a distance $s \sim j_hBt^2/\rho$ in a time t. In more convenient units,

$$s_{\mu m} \approx 0.1 I_{19} T_{MeV}^{-1} B_{10} \rho_{cgs}^{-1} t_{p\sec}^2 \qquad (1)$$

where s is in μm, I_{19} is the hot electron energy flux I_h in 10^{19}Wcm^{-2}, T_{MeV} is the hot electron T_h energy in MeV, B_{10} is the magnetic field in units of 10MG, ρ_{cgs} is the thermal plasma density in gm cm^{-3} and t_{psec} is the time in psec. For sub-psec pulselengths, little ion motion is expected unless the density is much less than solid or the magnetic field is much greater than 10MG, but considerable ion motion can take place in 10psec.

For a complete description of streaming instabilities in FI conditions, we need to include three particle species: hot electrons, thermal electrons and thermal ions. The three species are coupled by electromagnetic fields and by collisions. Ideally we would solve the Vlasov-Fokker-Planck (VFP) equations for the electrons and the VFP or fluid equation for ions, but we can represent the system adequately with separate, but coupled, fluid momentum equations:

$$\frac{d\mathrm{v}_\alpha}{dt} = \frac{q_\alpha}{m_\alpha}\mathrm{v}_\alpha \times B + \frac{q_\alpha}{m_\alpha}E - \nu_{\alpha i}\left(\mathrm{v}_\alpha - \mathrm{v}_i\right) - \frac{\nabla P_\alpha}{\rho_\alpha} \qquad (2)$$

where α = i,e,h for ions, thermal electrons or hot electrons, and $\nu_{\alpha i}$ is the collision frequency between species α and ions. P_α is the pressure of species α. For a filamentation instability, the relevant pressure is that transverse to the filament. Without a zeroth order magnetic field, and in the absence of collisions and resistivity, the resulting dispersion relation is

$$\left[\omega^2 - k^2c^2 - \omega_{pe}^2 - \sum_\alpha \frac{\omega_{p\alpha}^2 k^2 \mathrm{v}_{\alpha 0}^2}{\omega^2 - k^2 c_{s\alpha}^2}\right]\left[1 - \sum_\alpha \frac{\omega_{p\alpha}^2}{\omega^2 - k^2 c_{s\alpha}^2}\right] - \left[\sum_\alpha \frac{\omega_{p\alpha}^2 k \mathrm{v}_{\alpha 0}}{\omega^2 - k^2 c_{s\alpha}^2}\right]^2 = 0 \quad \text{where} \quad c_{s\alpha}^2 = \frac{\partial P_\alpha}{\partial \rho_\alpha} \qquad (3)$$

where, for species α, $\omega_{p\alpha}$ is the plasma frequency, $\nu_{\alpha 0}$ is the zeroth order mean velocity, and $c_{s\alpha}$ is the sound speed. In the limit of small $c_{s\alpha}$, ions are stationary, the second term is zero due to the requirement of zero total current, and the dispersion relation simplifies to a Weibel-related filamentation instability with a growth rate $k\langle\nu_0^2\rangle^{1/2}$ where $\langle\nu_0^2\rangle$ is the mean square drift velocity averaged over all electrons, both hot and thermal. Since the hot electrons constitute only a small fraction of the total, $|\omega^2|$ is much smaller than $k^2c_{s\alpha}^2$ unless the hot electron current is beamed with a very small divergence. Hence $k^2c_{s\alpha}^2$ dominates the denominators and the dispersion relation is changed from straightforward WI.

In the limit of very long wavelength and small ω, all three species are coupled to each other by electromagnetic fields or by collisions and there is no occasion for streaming instabilities. However, there is an intermediate regime in which low energy thermal electrons and ions are coupled together, but the hot electrons, if much more energetic, are relatively free to move with respect to the thermal plasma.

This regime is well known in the physics of cosmic rays (CR). Energetic CR have a Larmor radius many orders of magnitude greater than the Larmor radius of either thermal electrons or thermal ions. For wavelengths intermediate between the thermal and CR Larmor radii, an instability grows which is driven by the electric current j_h carried by CR as they stream through the thermal plasma. Unlike WI, the instability cannot generate a magnetic field where there is none initially. The CR are subject to a j×B force as they stream across the zeroth order magnetic field. The thermal plasma is set in motion by the opposite reactive – j×B force. The motions stretch the magnetic field which is frozen into the thermal plasma. As the field is stretched, the total magnetic field is increased, which in turn increases the j×B forces, and positive feedback produces instability. The instability can be analysed starting from the same momentum equations set out above for the Weibel instability. In its simplest form, the electron and ion thermal momentum equations are combined into one equation because the ions and electrons are strongly coupled. The equations describing the instability are

$$\rho \frac{dv_i}{dt} = -j_h \times B \qquad \frac{\partial B}{\partial t} = \nabla \times (v_i \times B) \tag{4}$$

Thermal pressure forces and magnetic pressure and tension forces can be neglected in most circumstances because the CR current density is very large. For the same reason, the CR are relatively unaffected by the magnetic field and j_h can be treated as uniform and constant. The equation for magnetic field growth arises from the ideal MHD Ohm's law which expresses magnetic flux freezing. Linearising these equations gives an instability growth rate

$$\gamma = \left(\frac{kBj_h}{\rho}\right)^{1/2} \tag{5}$$

Rearranging the equation for the growth rate gives $k^{-1} = j_h B \gamma^{-2}/\rho$, which corresponds to the equation ($s \sim j_h B t^2/\rho$) given above as a measure of the importance of ion motion. The difference here is that the displacement is growing exponentially rather than quadratically. As shown by Bell (2004, 2005)[1,2], the natural form of the instability is an expanding spiral of magnetic field encompassing a density cavity. Even if the instability grows from noise on a small spatial scale, it adopts a spiral and the instability grows by expansion of the spiral.

The same three fluid momentum equations set out above include the Weibel instability, the resistive filamentation instability and the CR streaming instability. Because of the transverse pressure and the non-negligible ion Larmor radius, neither the Weibel instability nor the CR steaming instability apply in their simplest forms. Laser-generated hot electrons sit in an intermediate regime, and we show now that despite the lack of thermal ion magnetization, laser-plasmas are subject to an instability very similar to the CR streaming instability.

We begin by dispensing with the hot electron momentum equation and assuming that the hot electron current is imposed by the hot electron source at the solid surface.

The momentum equations for the thermal electrons and ions can be combined into an Ohm's law

$$E = -v_i \times B + j_e \times B / n_e e - \eta j_e - \nabla P_e / n_e e \tag{7}$$

where j_e is the thermal electron current in the ion rest frame, and an ion momentum equation

$$\rho_i \frac{dv_i}{dt} = -j_h \times B - \nabla(P_e + P_i) - \frac{1}{\mu_0} B \times (\nabla \times B) \tag{8}$$

The ion momentum equation is supplemented by an energy equation which includes thermal conduction and resistive heating. These are coupled to the Maxwell equations

$$\nabla \times B = \mu_0 (j_e + j_h) \qquad \frac{\partial B}{\partial t} = -\nabla \times E \tag{9}$$

The retention of electron-ion collisions introduces diffusion of magnetic field and the resistive source of magnetic field $-\nabla \times (\eta j_e)$. This latter term provides a means of generating magnetic field where there is initially none.

Equations (7), (8), (9) and the associated energy equation are solved numerically in one dimensional cylindrical geometry as a function of radius r. A hot electron current j_h is imposed parallel to the axis.

$$j_h = \frac{I_h}{T_h} \left\{ f \cos^2\left(\frac{\pi}{2} \min\left(1, \frac{r}{r_{h1}}\right)\right) + (1-f) \cos^2\left(\frac{\pi}{2} \min\left(1, \frac{r}{r_{h2}}\right)\right) \right\}$$

j_h is fixed and does not change during the calculation. Figure 1 displays the results after 1 and 10 psec respectively for $I_h = 10^{18}$ Wcm^{-2}, $T_h = 300$ keV, $r_{h1} = 2$ µm, $r_{h2} = 10$ µm, $f = 2/3$. Initially, the background plasma has $\rho = 1$ gm cm^{-3} and $T_e = 100$ eV. The Spitzer resistivity and thermal conductivity are used with $A/Z = 2$. Magnetic field generation due to the variation in j_h occurs throughout the calculation although growth due to current gradients is opposed, particularly at early time, by variations in resistivity due to its dependence on temperature. The $-j_h \times B$ force accelerates the plasma away from the axis. By $t = 1$ psec this has made little difference to the density

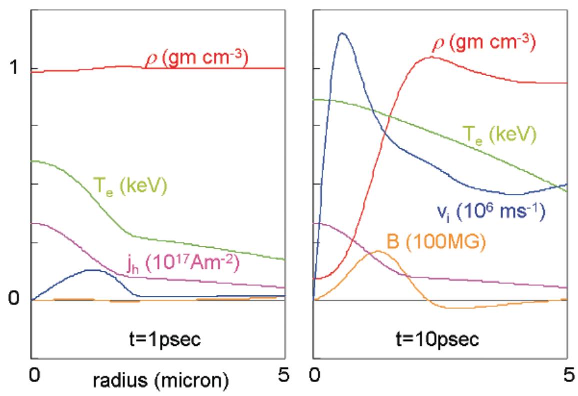

Figure 1. Profiles at t=1 psec and t=10 psec. f = 2/3. Normalisations are given in brackets.

profile, but by 10 psec an expanding density cavity has formed around the axis. Despite magnetic diffusion, the magnetic field is advected away from the axis leaving a partial magnetic cavity.

When f=2/3, only 20% of the total current and the hot electron energy flux is carried inside the radius r_{h1}=2 µm. If instead, we set f=1, while leaving the hot electron energy flux on axis I_h=10^{18} Wcm^{-2} unchanged, the gradients in j_h are larger, plasma temperature does not increase as rapidly, and the effects seen in figure 1 are enhanced as shown in figure 2, even though the total hot electron input is reduced ninefold. Comparison of the profiles of ρ and B with that for j_h show that the magnetic spiral and associated structures expand in time even though the driving current is localised near the axis. Because of the lower temperature, a snow-plough shock forms at the outer radius of the structure.

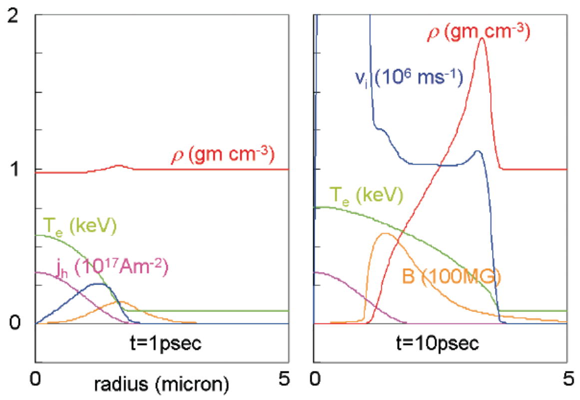

Figure 2. As for fig. 1, but with f = 1.

This analysis demonstrates that ion motion must be considered for laser pulses lasting 10 psec at solid density. An instability, associated with that known for streaming cosmic rays, causes the growth of expanding density cavities and magnetic spirals. The results are sensitive to thermal and magnetic diffusion, and a full VFP treatment is desirable.

References

1. A. R. Bell, MNRAS **353**, 550 (2004).
2. A. R. Bell, MNRAS **358**, 181 (2005).
3. A. Bret *et al., Phys Rev Lett*. **100**, 205008 (2008).
4. L. Gremillet, G. Bonnaud, F. Amiranoff, *Phys Plas*. **9**, 941 (2002).
5. Norreys, private communication (2008).
6. Y. Sentoku *et al*., *Phys Rev Lett.* **90**, 155001 (2003).
7. M. Tzoufras *et al*., *Phys Rev Lett.* **96**, 105002 (2006).