Vlasov-Fokker-Planck modelling of magnetic field generation by anisotropic pressure in laser-plasma interactions

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Introduction

The electric fields that occur to balance electron pressure gradients and maintain quasineutrality in collisional plasmas generally lead to magnetic field generation. In local equilibrium, magnetic field generation can occur by the well-known thermoelectric mechanism [1,2], $B \propto \nabla T_e \times \nabla \ln n_e$, in the presence of gradients in *both* electron temperature, T_e, and number density, n_e.

The magnetic fields that can develop in such plasmas can be significant [1,3], in that their presence affects the magnitude and direction of the particle fluxes, e.g. electron heat flux, and therefore the long time evolution of the system. This evidently has consequences for inertial fusion energy applications, as the coupling of the laser beams with the walls or pellet and the development of hot spots are all critical to the uniformity of the implosion.

In non-equilibrium, laser heated, scenarios, a number of other magnetic field generating mechanisms can arise even in the absence of density gradients. Non-local fluxes, arising when the gradients in the macroscopic quantities are small compared with the mean-free-path of a thermal electron, can result in magnetic field generation [4]. The spatially dependent distortion of the electron distribution function due to inverse bremsstrahlung absorption of laser momentum [5] and anisotropic pressure generated by the electron quiver in the laser field [6] can also be sources of significant magnetic fields.

Magnetic fields generated by laser speckles

Simulations were run of a single, linearly polarized laser speckle in an underdense plasma, using the Vlasov-Fokker-Planck code IMPACTA. The heating profile was a Gaussian spot of *1/e* radius 5 µm, with *x* polarization and with $I\lambda^2 = 1 \times 10^{15}$ W cm^{-2} µm^2. The plasma was a homogeneous CH plasma with an electron temperature of $T_e = 1$ keV, and with an electron number density of $n_e = 1 \times 10^{21}$ cm^{-3}.

As can be seen in figure 1, magnetic fields are generated in the plasma around the laser spot, due to the combined effects of anisotropic electron distribution function and non-local electron fluxes. Although the laser spot is circularly symmetric, the linear polarization (horizontal with respect to the pictures) results in the generation of anisotropic pressure.

The field structure is a quadrupole field. The shape of this quadrupole evolves with time due to the combined effects of (non-local) electron transport, and feedback of the magnetic field in rotating the anisotropic pressure. These result in a magnetization of the plasma in less than 50 ps, as shown in figure 2.

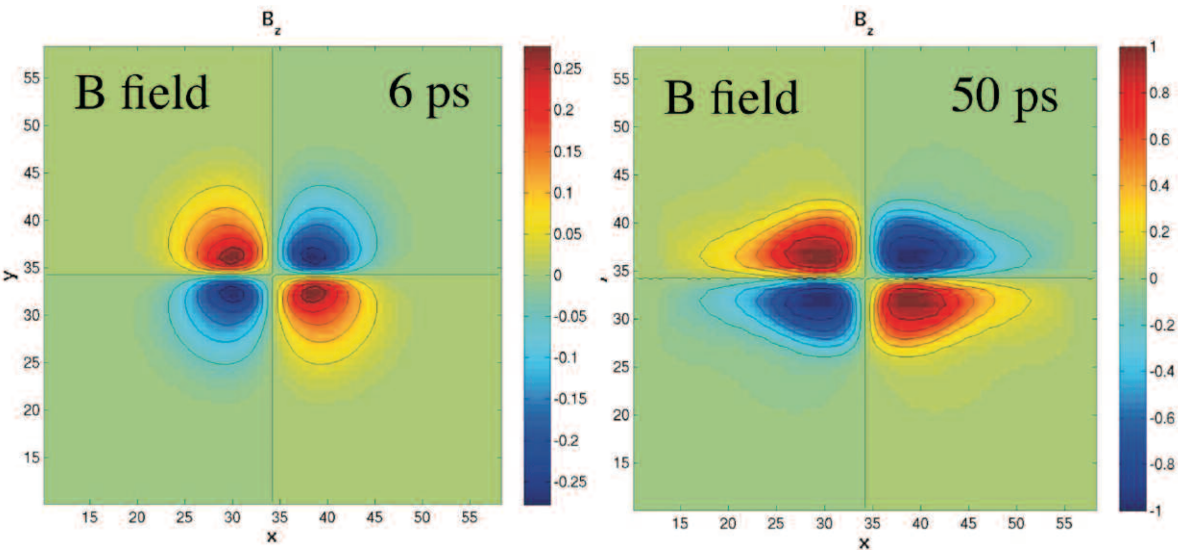

Figure 1. The magnetic field structure at various times.

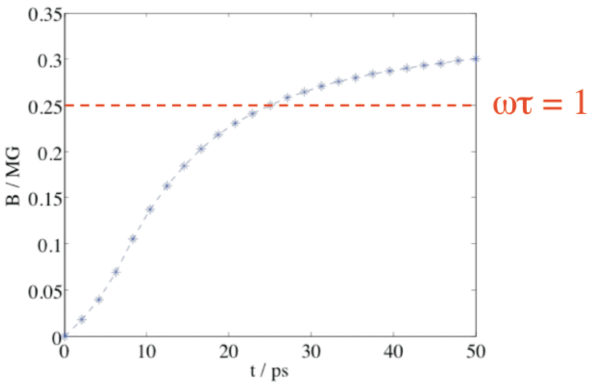

Figure 2. The magnetic field magnitude as a function of time.

The effect of this anisotropy and the magnetic fields on the transport is dramatic. In figure 3, the scalar heat flux is shown at the end of 50 ps. From the circular heating profile, an azimuthally symmetric heat flow would be expected, without the effects of magnetic field or anisotropic pressure. Here the heat flow has a strong angular dependence, and this results in redistribution of internal energy in a very non-uniform manner, as shown in the non-symmetric temperature distribution after 50 ps. This also affects the hydrodynamics of the system, as the gradients then evacuate the ions non-symmetrically.

The Vlasov-Fokker-Planck equation and magnetic field generation

The Vlasov-Fokker-Planck (VFP) equation is a conservation equation in 6D phase-space, with a smooth electron distribution function *f* representing a statistical average of the particles within a differential volume element. It includes the effect of both the conservation of *f* in the presence of macroscopic electric, **E**, and magnetic, **B**, fields, and the small angle Coulomb deflections of the underlying inter-particle interactions.

$$\frac{\partial f}{\partial t} + \mathbf{v}\cdot\frac{\partial f}{\partial \mathbf{r}} + \frac{q_j}{m_j}\left[\mathbf{E}+\mathbf{v}\times\mathbf{B}\right]\cdot\frac{\partial f}{\partial \mathbf{v}}$$
$$= -\frac{\partial}{\partial \mathbf{v}}\cdot\left\{f\langle\Delta\mathbf{v}\rangle\right\} - \frac{\partial}{\partial \mathbf{v}}\frac{\partial}{\partial \mathbf{v}}:\left\{f\langle\Delta\mathbf{v}\Delta\mathbf{v}\rangle\right\}$$

To relate this equation to meaningful physical quantities, velocity moments of the distribution function must be taken, which in general are given by:

$$M_l^m = \int_{-\infty}^{\infty}\{\mathbf{v}\}^l v^m f(t,\mathbf{r},\mathbf{v})d\mathbf{v}$$

where $v = |\mathbf{v}|$, and $\{\mathbf{v}\}^1$ denotes the tensor outer product of **v**, applied *l* times. By applying these generalized moments to the VFP equation, a chain of transport equations occurs, relating a particular moment of order *l* to moments of order *l+1* and *l–1*, which for electrons is given by:

$$\frac{\partial M_l^m}{\partial t} + \nabla\cdot M_{l+1}^m + \frac{e}{m_e}\left[\mathbf{E}\cdot\left(l\underline{\underline{I}}M_{l-1}^m + mM_{l+1}^{m-2}\right) - \mathbf{B}\times\left(l\underline{\underline{I}}M_l^m + mM_{l+2}^{m-2}\right)\right]_l$$
$$= \left\langle\left(C_{ei}+C_{ee}\right)\{\mathbf{v}\}^l v^m\right\rangle + H_{IB}$$

where C_{ei} and C_{ee} are the electron-ion and electron-electron collision operators, and a heating operator, H_{IB}, representing the change in the electron distribution function due to inverse bremsstrahlung absorption has now been included. The electron-ion collision operator can be approximately expressed in the Lorentz form, for a single ion species background, as:

$$C_{ei} = \frac{A}{v^3}\frac{\partial}{\partial \mathbf{v}}\cdot\left[\left(v^2\underline{\underline{I}} - \mathbf{v}\mathbf{v}\right)\cdot\frac{\partial f}{\partial \mathbf{v}}\right]$$

where $A = Zn_ie^4 \ln \Lambda/8\pi\varepsilon_0^2m_e^2$, Ze and n_i are the ion charge and density, and ln Λ is the Coulomb logarithm. Hence this term disappears in the *l=1*, *m=3* transport equation, leading to a somewhat generalized form of Ohms law:

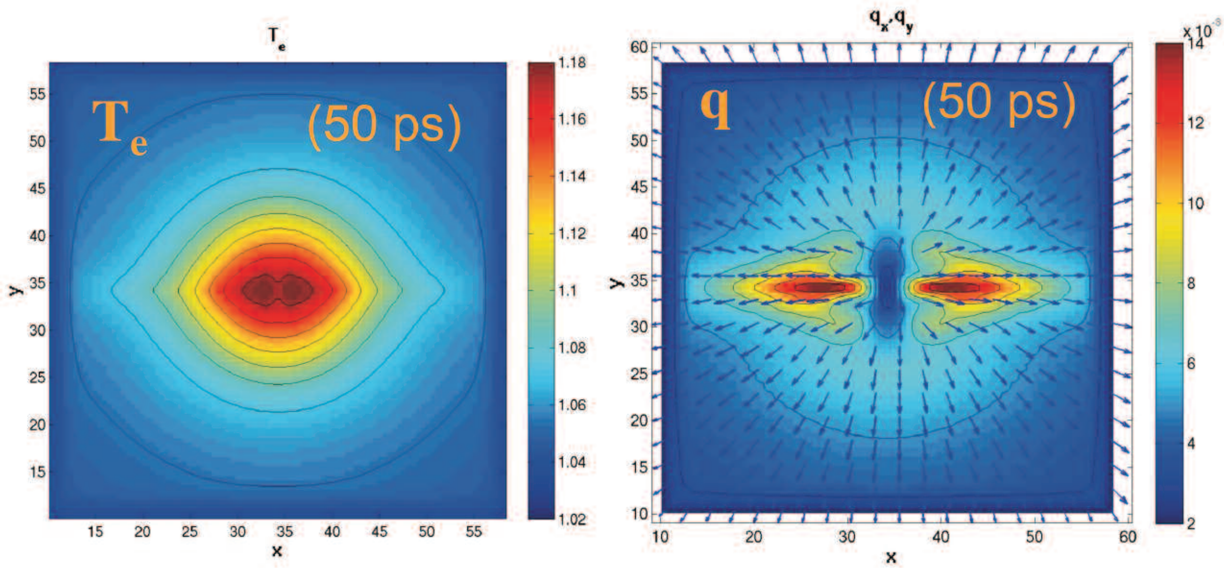

Figure 3. The electron temperature and scalar heat flux after 50 ps laser heating.

$$\frac{\partial M_1^3}{\partial t} + \nabla \cdot M_2^3 + \frac{e}{m_e}\left(\mathbf{E}M_0^3 + 3\mathbf{E}\cdot M_2^1\right) - \omega_c \times \left(M_1^3 + 3M_3^1\right) = \left\{\Delta M_{ee}\right\}_1^3$$

Note that the heating operator has no time-averaged effect on vector moments. The term involving the cyclotron frequency, ω_c, is responsible for diffusion and advection of the fields (i.e. the Nernst effect [2]). The other terms can be sources of magnetic fields, but the most important is the second term. This includes the thermoelectric, non-local and anisotropic pressure magnetic field generating mechanisms. This can be seen by isolating the relevant terms, i.e. the second and third terms, and rearranging:

$$\mathbf{E} = -\frac{m_e}{2e}\frac{\nabla \cdot M_2^3}{M_0^3}$$

where for simplicity, the traceless part of the moment in the electric field term has been neglected. Now, if this equation is split into traceless and non-zero trace parts, the latter yields an electric field:

$$\mathbf{E} = -\frac{m_e}{6e}\frac{\nabla M_0^5}{M_0^3}$$

which leads to non-local and thermoelectric magnetic field generation through Faraday's law [4]. The traceless part leads to magnetic field generation due to electron pressure anisotropy. To capture all these effects, which are intricately coupled to each other, it is best to solve the VFP equation combined with Maxwell's equations.

IMPACTA

The code IMPACTA solves the VFP equation in two Cartesian spatial dimensions. The angular information of the velocity 3-space is expanded as a series of Cartesian tensors [7], related to spherical harmonics, which are eigenfunctions of the collision operators. In IMPACTA, the series is truncated for rank 2 tensor objects, which is to say that the effects of anisotropic pressure are retained. It is an extension of the code IMPACT, described in [8].

In addition, ions are represented by a hydrodynamic background, so that the expanded VFP equations for the electrons is cast in the stationary frame of the ion fluid. The code integrates the finite difference equations with an implicit algorithm, which allows nanosecond timescale physics to be studied. Inverse bremsstrahlung heating is represented by a modified version of the Langdon operator [5] to include the effect of the absorption on anisotropic pressure [9].

Conclusions

The fast growth of strong magnetic fields has been demonstrated, under fusion relevant conditions. Laser speckles in the interaction with sub-critical plasma can generate these fields, due to the development of electron distribution function anisotropy resulting from oscillations in linearly polarized electromagnetic radiation.

These magnetic fields and anisotropic distributions lead to a spatially dependent inhibition of electron and ion transport. This affects the long-term evolution of the system, and may need to be considered in modelling of fusion related scenarios.

Acknowledgements

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References

1. J. A. Stamper *et al.*, *Phys. Rev. Lett.* **26**, 1012 (1971).
2. M. G. Haines, *Can. J. Phys*. **64**, 912 (1986).
3. J. A. Stamper and B. H. Ripin, *Phys. Rev. Lett*. **34**, 138 (1975); A. Raven, O. Willi, and P. T. Rumsby, *Phys. Rev. Lett*. **41**, 554 (1978); M. Borghesi *et al.*, *Phys. Rev. Lett*. **81**, 112 (1998).
4. R. J. Kingham and A. R. Bell, *Phys. Rev. Lett*., **88**, 045004 (2002).
5. A. J. Langdon, *Phys. Rev. Lett*., **44**, 575 (1980).
6. B. Dubroca, M. Tchong, P. Charrier, V. T. Tikhonchuk and J.-P. Morreeuw, *Phys. Plas.,* **11**, 3830 (2004).
7. T. W. Johnston, *Phys. Rev*., **120**, 1103 (1960).
8. R. J. Kingham, *Comp. Phys.*, **194**, 1 (2002).
9. Maximov and Silin, *Sov. Phys. JETP* **76**, 39 (1993).