Impedance matching – can it tell us anything about fast electron transport?

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Introduction

The absorption of energy from very intense CPA laser beams, such as occurs in Fast Ignition, leads to electron 'beams' with many extreme properties[1] and often counter-intuitive behaviour. The concept of an impedance associated with the electron beam gives some structure to these properties.

Absorption from relativistically intense laser beams

A simple argument based on energy balance leads to some of the fundamental properties of these electron beams, momentum balance also involves the ion motion but because of the large ion/electron mass ratio the energy flow is almost entirely into electrons. In the case of $a_0 > 1$ we can approximate the ponderomotive energy of the electrons as a_0mc^2 and the relativistically corrected critical density as

$$n_1 = a_0 n_c = a_0 \frac{m\varepsilon_0\omega^2}{e^2}$$

The maximum energy flux that can be carried away by electrons of this energy and density, all moving at speed c is

$$P = n_1 a_0 m c^3 = \varepsilon_0 E^2 c = \frac{E^2}{Z_0}$$

where we have used the relations $c = (\mu_0\varepsilon_0)^{-1/2}$ and $Z_0 = (\mu_0\varepsilon_0)^{1/2}$ is the impedance of free space with the numerical value 377 Ohm.

This is to be compared with the energy flux in the original laser beam given by the Poynting vector $(\mathbf{E} \times \mathbf{H}) = E^2/Z_0$ (*cf* $P=V^2/Z$ in an electrical circuit).

This apparent coincidence of the energy flux in the electro-magnetic wave and the energy carried by electrons at the corrected critical density is in fact a necessary consequence of the fact that the group velocity of the wave drops to zero at this density showing that all the wave energy is being used to polarise the medium.

100% absorption is impossible unless the electrons have an energy greater than the ponderomotive energy since consideration of charge build-up shows that there must be a return current almost exactly matching the current carried by energetic electrons. Even at 50% absorption the velocity of the returning electrons will be relativistic.

Impedance of the laser driven fast electron source

For the ponderomotive scaling the 'voltage' associated with the electron source is $V = \Phi/e = a_0mc^2/e = Ec^2/\omega$

If the focal spot radius is r then incident power is $E^2\pi r^2/Z_0$

An absorption fraction f implies $VI = f\, E^2\pi r^2/Z_0$ and the impedance $Z=V/I$ is given by

$$Z = \frac{Z_0}{\pi f}\frac{(c/\omega)^2}{r^2}$$

Typically $Z = 0.5$ Ohm for a 1 μm laser, a 7 μm focal spot and 50% absorption. The large disparity between Z and Z_0 is due to the large area of the focal spot compared to the square of the skin depth c/ω.

Impedance associated with the Alfven current

The current carried by the laser driven electron beam is frequently compared to the Alfven current despite the fact that the derivation of the Alfven limit is entirely inappropriate to these conditions. The derivation of the Alfven current requires plasma which is charge neutral but carries no neutralising current so that magnetic fields may be calculated from the distribution of beam current. In the laser plasma case there is almost complete current neutralisation1), at least at early times, before Weibel-like instabilities have possibly caused spatial separation of the forward and return currents.

Davies[2] gives a convenient form for the Alfven current $I_A = 4\pi p/(\mu_0 e)$ for electrons with momentum p. In the relativistic limit we can take the electron energy $E = eV = \gamma mc^2$ so that $p = eV/c$ and $I_A=4\pi V/\mu_0 c$ where V is now the voltage associated with the Alfven current so $Z = V/I = Z_0/4\pi \sim 30$ Ohm.

In the presence of strong filamentation and current separation one would expect the electron beam of overall impedance 0.5 Ohm to break into ~ 60 filaments each with impedance 30 Ohm. In 2D simulations[1] with only one transverse direction one might naively expect ~ (60)[1,2] filaments which is broadly in line with published data.

Coupling to transmission lines

The standard results for the impedance of transmission lines gives $Z = Z_0 w/d$ for parallel plates of width w and separation d and $Z = Z_0\ ln(r_2/r_1)$ for coaxial lines of radii r_1 and r_2. Clearly one can obtain $Z \sim 0.5$ Ohm only for large areas and small separations.

Interestingly one finds that there is large lateral flow of electron energy at front and rear of thin flat targets giving rise to large fields and ion emission [3] at very large distances (large w/d) from the focal spot. On the other hand attempts to couple absorbed energy into thin wires ($r_2/r_2 \sim 1$) show poor efficiency [4]. This comparison of geometries cannot be entirely attributed to impedance matching since magnetic deflection and $\mathbf{E} \times \mathbf{B}$ drifts encourage the flat plate spreading while inhibiting the coupling into thin wires.

Conclusions

Considerations of the electrical impedance associated with current flow in laser irradiated plasmas offer a unified view of diverse observations. The analogy must be applied with care to situations which are transient and have poorly defined boundaries.

References

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4. J. Pasley *et al.*, *Phys. Plasmas* **14**, 120701 (2007).