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Prolific pair production with high power lasers

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High-power laser facilities have made dramatic progress recently, and the next few years may bring intensities of 10^{23}-10^{24} Wcm^{-2} within reach. This naturally opens up new physics regimes (Koga *et al.* 2006[6], Müller *et al.* 2008[8]). The relativistic Lorentz factor of an electron oscillating in vacuum in the electromagnetic field of a planar linearly polarised laser beam is 860 $(I_{24}\lambda_{\mu m}^2)^{1/2}$ where I_{24} is the laser intensity in 10^{24} Wcm^{-2} and $\lambda_{\mu m}$ is the laser wavelength in micron. The corresponding peak electric and magnetic fields are $2.7 \times 10^{15}\, I_{24}^{1/2}$ Vm^{-1} and 91 $I_{24}^{1/2}$ GG. The Schwinger field $E_{crit} = 1.3 \times 10^{18}$ Vm^{-1} required for spontaneous electron-positron pair creation out of the vacuum would be attained at a laser intensity of 2.3×10^{29} Wcm^{-2} (Schwinger 1951[10], Salamin *et al.* 2006[9]). Here we show how copious pair production by accelerated electrons interacting with the laser field can be achieved using laser intensities ~10^{24} Wcm^{-2} (Bell & Kirk 2008[2]). The key is to exploit the large transverse electromagnetic field seen by an electron when it experiences laser beams that are not propagating in parallel. We illustrate this effect by computing the case of counter-propagating, circularly polarized beams. The advantage offered by this configuration is analogous to the dramatic increase in centre-of-mass energy when using colliding particle beams instead of stationary targets. This advantage remains in less specific configurations such as tight focus and reflection from a solid surface. Consequently, it may be possible to convert a large fraction of the laser energy into electron-positron pairs at a laser intensity of ~10^{24} Wcm^{-2} at approximately solid plasma density.

Relativistic electrons with Lorentz factor γ moving perpendicularly to a homogeneous magnetic field B produce pairs if $\gamma B/B_{crit}$ is greater than or of the order of unity, where $B_{crit} = 4.414 \times 10^4$ GG is the magnetic equivalent of the Schwinger field E_{crit}. Pair production occurs if the parameter

$$\eta = \frac{\gamma E \sin\vartheta}{E_{crit}}$$

is of order unity or larger, where θ is the angle between the electric field and the electron momentum.

Classically, the electron equation of motion, including radiation reaction according to the Landau & Lifshitz (1975)[7] prescription is

$$\frac{d(\gamma\hat{\mathbf{a}})}{dt} = -\frac{e}{mc}(\mathbf{E} + \hat{\mathbf{a}} \times \mathbf{B}) - \frac{2e^4\gamma^2}{3m^3c^5}\hat{\mathbf{a}}\left|\mathbf{E} + \hat{\mathbf{a}} \times \mathbf{B}\right|_\perp^2$$

The terms that have been omitted here are of order γ^{-2}. The final term of the above equation represents the drag and energy loss due to radiative emission, to which pair production is related, and is proportional to the square of that component of the Lorentz force perpendicular to the direction of motion. In the case of a planar uni-directional wave, **E** and $\beta \times \mathbf{B}$ nearly cancel to zero in the laboratory frame.

Two counter-propagating laser beams produce a standing wave with nodes at which $B = 0$ and the electric field rotates in direction with constant amplitude. By symmetry, an electron placed exactly at the node does not move in the direction of the waves, but performs circular motion with the centripetal force provided by the electric field. The equation of motion then simplifies to

$$\frac{d(\gamma\hat{\mathbf{a}})}{dt} = \frac{eE}{mc}\left\{-\mathbf{i} - \frac{2\gamma^2\hat{\mathbf{a}}}{3}\frac{E}{E_{cl}}\left[1 - (\hat{\mathbf{a}}.\mathbf{i})^2\right]\right\}$$

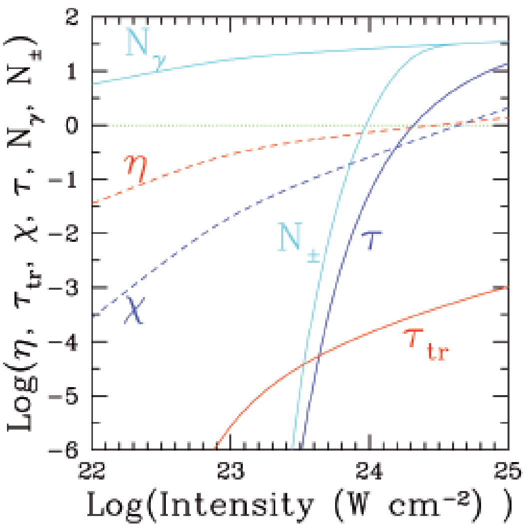
where **i** is the unit vector in the direction of the electric field and we have defined the characteristic value E_{cl} of the electric field in classical electrodynamics: $E_{cl} = m^2c^4/e^3 = E_{crit}/\alpha_f$ (α_f is the fine-structure constant). As can be seen from the equation of motion in this form, the radiation reaction force becomes important when $\gamma^2 E/E_{cl}$~1, i.e., η~$(\gamma\alpha_f)^{-1}$, a situation that is reached at laser intensities ~$10^{23}\ \lambda_{\mu m}^{-4/3}$ Wcm^{-2}. The Landau & Lifshitz prescription for the radiation reaction term is valid up to $\eta \approx 1/\alpha_f$ (di Piazza 2008[3]) but quantum effects already intervene at $\eta \sim 1$ (Erber 1966[4]).

Within a small fraction of a laser period, the electron trajectory adjusts itself such that the component of electric field parallel to the direction of motion precisely compensates the radiative losses, whilst the perpendicular component enforces circular motion at the laser period, with Lorentz factor $\gamma = a\sin\theta$ where $a = eE\lambda/2\pi mc^2$ is the laser strength parameter. Combining these, we can express E in terms of η. In an underdense plasma, the total electric field E is related to the intensity I of each laser beam (separately) by $I = cE^2/16\pi$, which gives

$$I_{24} = 2.75\eta^4 + 0.28(\eta/\lambda_{im})$$

This relation is plotted in the figure for a laser of wavelength 1μm. In terms of these parameters, $\sin\theta = 0.53\eta^{1/2}(I_{24}\lambda_{\mu m})^{-1/2}$. At low intensity, the particle moves almost exactly perpendicularly to E and η rises linearly with the laser intensity. However, when radiation reaction becomes important, this rise is slowed, and $\eta=1$ is not achieved until $I_{24}=3$. The photons radiated because of the acceleration of the electron in the electric field of the laser, which we term curvature radiation but is also known as bremsstrahlung (Erber 1966 [4]), can be described classically using the theory of synchrotron radiation. This predicts that most radiated photons are emitted with an energy $h\nu_s = 0.44\eta\gamma mc^2$ where $\gamma mc^2 = 328(\eta\lambda_{\mu m})^{1/2}$ MeV is the energy of the relativistic electron. Because of quantum effects analogous to the Klein-Nishina corrections to the Thomson cross-section (Erber 1966 [4], Aharonian 2004 [1]), the radiative energy loss does not proceed in the continuous manner implied by the equation of motion when $\eta>1$ and $I_{24}>>1$. Nevertheless, the classical trajectory is an adequate approximation in the intensity range 10^{23}-10^{24} Wcm^{-2}, which is of interest here, since the photon energy is significantly less than the electron energy.

Other important quantum effects are already present at intensities in the range 10^{23}-10^{24} Wcm^{-2}. There are two processes that produce electron-positron pairs. At low laser intensities, the trident process dominates, in which an electron produces an electron-positron pair via an intermediate virtual photon. In a homogeneous electric or magnetic field (a good approximation when $\lambda_{laser} >> h/mc = 2.4 \times 10^{-6}$ μm) the rate is given by Erber (1966 [4]) and Urrutia (1978) [11]. Expressed as a production rate per electron per laser period, it can be written $\tau_{tr} = 0.06\,(I_{24}\lambda_{\mu m}^2)^{1/2}\eta^{1/4}\exp(-8/(3\eta)^{1/2})$ for $\eta<<1$. For $\eta>1$, it goes over to a slow logarithmic increase. The precise form is plotted in the figure.



Plotted as a function of laser intensity for a laser wavelength of 1 μm, are the parameters η and χ, the optical depth τ of the curvature photon across one wavelength, the number $N_{\pm}$ of pairs produced per electron in one laser period by both this process and by that of pair production by the trident process (labelled τ_{tr}), and the number N_γ of curvature radiation photons produced per electron per laser period.

At higher intensities, the related process becomes important, in which the electron first produces a real photon by curvature radiation, which subsequently creates a pair. The photon absorption coefficient is controlled by the parameter

$$\chi = \frac{h\nu_s E}{2mc^2 E_{crit}}$$

From the expression given above for $h\nu_s$ and writing E in terms of I_{24}, $\chi = 0.42\eta^{3/2}(I_{24}\lambda_{\mu m})^{1/2}$ as plotted in the figure. The photon optical depth to absorption in a path length λ_{laser} is $\tau = 12.8\,(I_{24}\lambda_{\mu m}^2)\exp[-4/(3\chi)]$, for $\chi<<1$, peaking at $\chi \approx 8$ and falling off for larger χ (Erber 1966 [4]), also as plotted in the figure. We estimate the total pair-production rate per electron per laser period as the product of the photon absorption probability 1-exp(-τ) in a length λ_{laser} multiplied by the rate of production of photons by curvature radiation. This quantity, together with the number of curvature radiation photons emitted per electron per laser period (i.e. the energy radiated divided by $h\nu_s$), $N_\gamma = 6.42\alpha_f\gamma$, is also shown in the figure.

Inspection of this figure shows that for laser intensities less than roughly $I = 3.3 \times 10^{23}$ Wcm^{-2} where $\eta = 0.51$, pair production is dominated by the trident process. At this intensity, each electron in the zone where the laser beams overlap produces on average 3×10^{-5} pairs in a single laser period. The curvature radiation energy losses, which are 123 kW per electron, dominate over pair production. They are sufficient to damp the laser beams in 1.8 n_{23}^{-1} fsec where n_{23} is the electron density in 10^{23} cm^{-3}. The total number of pairs produced in the absence of other energy losses is 7×10^4 per Joule of laser energy.

At intensities above $I = 3.3 \times 10^{23}$ Wcm^{-2} the number of pairs produced by photon-induced pair production rises steeply. These pairs are also accelerated and generate additional photons and pairs. A cascade should develop when $N_{\pm} \approx 1$, which occurs at $I_{24} \approx 1$ and $\eta \approx 0.7$. At this intensity, the laser power should be divided roughly equally between photons and pairs with energy ~80 MeV per photon and per pair. Complete conversion of laser energy to photons and pairs implies the production of ~4×10^{10} pairs per Joule of laser energy. The precise conditions under which a cascade is initiated are, however, sensitive to geometrical effects related to the intersection angle and the intersection volume of the laser beams.

More comprehensive calculations (Kirk *et al.* 2009 [5]) extend the above analysis to linear polarization and realistically shaped laser pulses. A statistical average over possible initial electron positions shows that the above results are approximately applicable in broad terms to a range of counter-propagating laser beams and, by inference, to incident and reflected beams in laser-solid interactions. We predict that pair-production should be a standard feature of laser-plasma interactions at intensities in the range 3×10^{23}-10^{24} Wcm^{-2} at a laser wavelength of 1 μm.

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