Weak collisionless shocks in laser plasmas

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Abstract

We have previously described an analytic model for the structure of steady state shock like structures in a collisionless plasma and showed how these might be relevant to observations of localized high electric fields in compressed pellets and to some experiments on ion acceleration. Here we present some more general results on these structures, describing the parmater range within they exist and some information on how the structure varies within this parameter range.

1 Introduction

In last year's report we described a simple analytic theory of weak collisionless shocks in an unmagnetized collisionless plasma [1, 2] and argued that it may be relevant to observations of localized high electric fields in laser compressed pellets [3–6] and also to production of ion beams with low energy spread [7]. Here, after a brief summary of the basic theory,we extend our earlier work by presenting some results on the scaling of some of the key quantities and discussing in more detail the way in which the structures depend on the parameters of the problem.

2 Theory

We look for nonlinear structures that are stationary in an appropriately chosen frame of reference within a collisionless plasma. The theory is very similar to the well-known theory of ion sound solitary waves, except that we include a finite ion temperature. The result is that some of the incoming ions are reflected, so that the solitary wave becomes an asymetrical structure more like a shock with a monotonic rise of the potential to a maximum value followed by an oscillatory potential on the downstream side.

For a single species plasma there are two dimensionless parameters that govern the existence of the structure and its nature in the domain where it does exist. These are T, the ratio of the electron temperature to the upstream ion temperature, and a Mach number M defined in terms of the approximate ion sound speed $\sqrt{\frac{T_e}{m_i}}$. This is not the true Mach number since it neglects ion pressure, but is convenient and, since the electron temperature needs to be substantially above the ion temperature for the structures to exist, it is never very far from the true Mach number. As discussed in our previous paper [1], a self consistent steady state solution can only be found within a limited range of these parameters.

By numerical trial and error we have estimated the upper and lower Mach number limits within which solutions exist for a range of values of the temperature ratio, with the result shown in Figure 1.
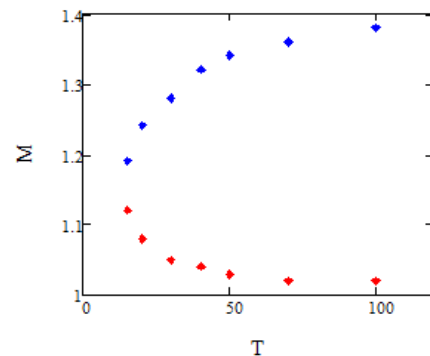

Figure 1. Upper (blue) and lower (red) limits of the allowed Mach number range, estimated by numerical trial and error for a range of electron/ion temperature ratios.

There is a minimum electron/ion temperature ratio of a little below 15, with the range of allowed Mach numbers broadening above this value. The existence of a fairly low upper Mach number for these structures is consistent with early computer simulations carried out by Forslund and Freidberg [8], who found structures with

the type of oscillatory behaviour we have found, involving a small number of reflected ions, up to a critical Mach number above which most ions were reflected and there was a population of trapped ions downstream.

For any given temperature ratio the fraction of ions reflected goes up with M. In Figure 2 we show the dependence of this fraction on T at the maximum allowed Mach number, the indication being that it levels off at a comparatively low value.
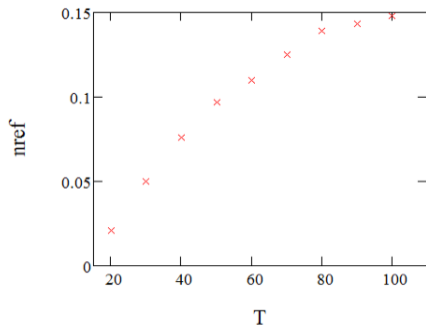

Figure 2. The reflected ion density as a function of temperature at the maximum Mach number.

Figure 3 shows the normalized potential maximum (in terms of T_i/Ze, with T_i the ion temperature, Z the ion charge number and e the electron charge) as a function of T, again at the maximum possible value of M, this time giving what appears to be a linear dependence.
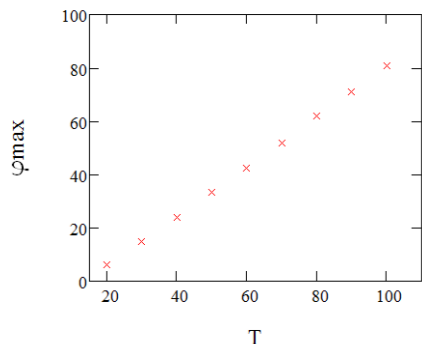

Figure 3. The normalized potential maximum as a function of T at the highest allowed vaue of M.

The slightly irregular nature of these graphs is the result of the maximum allowed Mach number being estimated by a process of trial and error.

The requirement for a high temperature ratio can be understood from Poisson's equation. A smooth potential ramp going from zero to a maximum value needs $n_e > n_i$ when ϕ is small then $n_e < n_i$ for larger ϕ, so as to give the correct curvatures. If T is too small n_e, which is assumed to take the thermal equilibrium form $\exp(\frac{e\phi}{T_e})$, increases too rapidly with ϕ for the latter condition to be possible. As T becomes large the lower Mach number limit approaches one, as might be expected since the

Mach number as we define it approaches the true Mach number in this limit. The existence of an upper Mach number limit is again connected with the need for a particular solution of Poisson's equation to exist. As the upper Mach number is approached the amplitude of the downstream oscillations decreases until very close to it there is essentially a constant potential downstream with $n_e = n_i$. Beyond this point the ions are slowed down too much and it becomes impossible to obtain a solution of the required form. As pointed out by Forslund and Freidberg the nature of the shock changes and we have not found any simple description of this. Some recent simulations have been carried out by Macchi et al [9], investigating the time dependence of these structures as energy is transferred from the field to the accelerated ions. Consideration of this time dependence is beyond the scope of our simple model which may be expected to be valid over suitably short time scales.

3 Conclusion

We have extended our earlier work on weak collisionless shocks that we suggest play a role in some important laser plasma phenomena. The results presented here give the parameter limits within which these structures exist and some general scalings. In future work we intend to continue our investigation of these structures and their relevance to problems in laser-plasma interactions.

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