Anisotropic cooling of electron beams interacting with intense laser pulses

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1 Introduction

Over the last few decades laser technology has advanced exponentially, regularly passing landmarks in the achievable intensities. Already, intensities exceeding 10^{22} W/cm^2 have been produced, and forthcoming facilities such as the Extreme Light Infrastructure (ELI) are expected to pass 10^{23} W/cm^2 in the next decade. At such intensities, qualitatively new physics will become experimentally accessible for the first time.

Many of the proposed experiments using these ultra-high power laser facilities will involve colliding electron beams with intense laser pulses to produce high frequency radiation through Compton backscattering. The quality of these secondary beams, such as their coherence and intensity, are highly sensitive to the properties of the electron beams, such as their average energy and momentum spread. These properties will vary as the electrons interact with the laser, so a proper understanding of their evolution is essential to ensure the production of high quality radiation.

At the intensities anticipated at ELI, both radiation reaction and quantum electrodynamical effects are expected to strongly affect an electron's motion. In this article, we explore how these effects alter the momentum spread of a relativistic electron beam as it passes through an intense laser pulse, and in particular how they affect the directionality of the resultant beam cooling. For further details and quantitative examples, see [1].

2 Classical and quantum radiation reaction

Before we can address the evolution of the bulk properties of an electron beam, we must understand how an individual particle interacts with electromagnetic fields. In most cases this is well understood: the particle accelerates according to the Lorentz force,

$$\ddot{x}^a = \frac{e}{m} F^a{}_b \dot{x}^b, \tag{1}$$

where $-e$ is the charge and m the mass of the particle, F is the electromagnetic field, and x is the particle's worldline, with overdots denoting differentiation with respect to proper time. However, the accelerations produced by the intense laser pulses of upcoming facilities are sufficiently great that (1) must be supplemented by an additional force describing the recoil due to emission of radiation. The exact form of this radiation reaction force remains contentious [2], but in the classical limit it is commonly believed to be well described by the Landau-Lifshitz equation [3],

$$\ddot{x}^a = \frac{e}{m}\left(F^a{}_b + \tau \dot{x}^c \partial_c F^a{}_b\right) \dot{x}^b + \tau \frac{e^2}{m^2} \Delta^a{}_b F^b{}_c F^c{}_d \dot{x}^d, \tag{2}$$

where $\tau = e^2/6\pi m$ is the 'characteristic radiation time' and $\Delta^a{}_b = \delta^a_b + \dot{x}^a \dot{x}_b$ is the $\dot{x}$-orthogonal projection operator.

At higher intensities (2) is no longer adequate, as quantum corrections cannot be ignored. The importance of quantum effects can be assessed from the 'quantum nonlinearity parameter',

$$\chi = \frac{e\hbar}{m^2} \sqrt{F^{ab} F_{ac} \dot{x}_b \dot{x}^c}, \tag{3}$$

which measures the electric field observed by the particle in units of the Sauter-Schwinger field $m^2/e\hbar$. When χ is too large for quantum effects to be ignored, but still significantly less than unity, the energy of a typical photon emitted is much less than that of the radiating particle. It follows that the primary effect of quantum dynamics in such conditions is to reduce the rate at which energy is emitted by an overall factor $g(\chi)$, depending on the quantum nonlinearity parameter. The function $g(\chi)$ is a nontrivial integral over Bessel functions; for illustrative purposes, we will use the approximation $g(\chi) = (1 + 12\chi + 31\chi^2 + 3.7\chi^3)^{-4/9}$ introduced in [4].

In the regime where the discreteness of photon emission (and pair production) can be neglected, then, we obtain a quantum-corrected classical equation of motion [5] by scaling the radiation reaction terms in (2) by $g(\chi)$:

$$\ddot{x}^a = \frac{e}{m}\left(F^a{}_b + \tilde{\tau} \dot{x}^c \partial_c F^a{}_b\right) \dot{x}^b + \tilde{\tau} \frac{e^2}{m^2} \Delta^a{}_b F^b{}_c F^c{}_d \dot{x}^d, \tag{4}$$

where $\tilde{\tau} = g\tau$. We use (2) and (4) as our starting point for an investigation into transverse and longitudinal beam cooling.

3 Transverse and longitudinal beam cooling

We are interested in the evolution of large-scale properties of an electron beam of tens of pC or more, containing some 10^8 particles. It would be impractical to follow the motion of each particle, and for our purposes unnecessary. Instead, we describe the beam by a distribution function $\mathcal{F}(x, \dot{x})$, which satisfies the Vlasov equation

$$\mathcal{L}_V(\mathcal{F}\Omega) = \left(\frac{d\mathcal{F}}{d\phi} + \beta\mathcal{F}\right)\Omega = 0. \tag{5}$$

Here, Ω is the phase space volume element and $\mathcal{L}_V$ is the Lie derivative with respect to the vector field V which represents the direction of increasing 'time' parameter ϕ. Since it is our intention to apply (5) to a laser pulse, it is convenient to take ϕ to be the phase of the pulse rather than proper time, and hence a lightlike (rather than timelike) parameter. The pulse itself we model as a plane wave,

$$\frac{e}{m}F^a{}_b = a_\epsilon(\phi)\left(k^a\epsilon_b - \epsilon^a k_b\right) + a_\lambda(\phi)\left(k^a\lambda_b - \lambda^a k_b\right) \tag{6}$$

where the functions a_ϵ and a_λ are dimensionless measures of the electric field strength in the ϵ and λ directions, respectively, and k is the null wave 4-vector.

For exploring beam cooling, the crucial quantity in (5) is β, the divergence of V, determined by $\mathcal{L}_V\Omega = \beta\Omega$. This determines the rate of contraction of phase space under the flow of V, and is related to the rate of change of entropy [6]. To analyse this further, we introduce velocity-space coordinates

$$u_\phi = -k\cdot\dot{x}, \quad u_\epsilon = \epsilon\cdot\dot{x}, \quad u_\lambda = \lambda\cdot\dot{x}. \tag{7}$$

(The fourth component of velocity is not an independent variable, but can be determined from the normalisation condition $\dot{x}^2 = -1$.) Then it follows from (5) that

$$\beta = \frac{\partial}{\partial u_\phi}\left(\frac{\mathcal{A}_\phi}{u_\phi}\right) + \frac{\partial}{\partial u_\epsilon}\left(\frac{\mathcal{A}_\epsilon}{u_\phi}\right) + \frac{\partial}{\partial u_\lambda}\left(\frac{\mathcal{A}_\lambda}{u_\phi}\right), \tag{8}$$

where the acceleration functions $\mathcal{A}_\phi$, $\mathcal{A}_\epsilon$ and $\mathcal{A}_\lambda$ are obtained by contracting the RHS of the relevant equation of motion (1), (2) or (4) with $-k$, ϵ and λ, respectively.

Our first observation is that, for electrons evolving according to (1), $\beta = 0$, so the beam cooling effect is entirely due to radiation reaction. Before turning to the cases of classical and quantum radiation reaction, we first note that β naturally decomposes into longitudinal and transverse parts,

$$\beta_\parallel = \frac{\partial}{\partial u_\phi}\left(\frac{\mathcal{A}_\phi}{u_\phi}\right), \quad \beta_\perp = \beta - \beta_\parallel. \tag{9}$$

The transverse velocities u_ϵ and u_λ can be integrated out of (5), yielding a longitudinal Vlasov equation for the reduced distribution $f(x, u_\phi) = \int \mathcal{F} du_\epsilon du_\lambda$, in which $\beta_\parallel$ assumes the role of β in the full Vlasov equation (5).

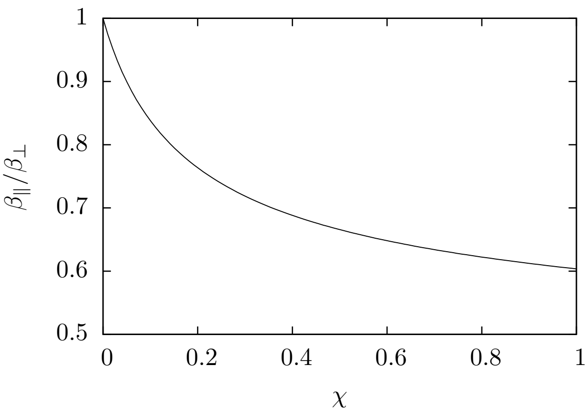

Figure 1: Ratio of longitudinal ($\beta_\parallel$) to transverse ($\beta_\perp$) phase space contraction, when the quantum reduction of radiated power is taken into account. For the classical case, this ratio is always unity.

We first consider the classical case, in which each electron behaves according to (2), and use carets to distinguish quantities from their equivalents in the quantum case. Here we find

$$\hat{\beta}_\parallel = \hat{\beta}_\perp = -2\tau\left(a_\epsilon^2 + a_\lambda^2\right)u_\phi \le 0. \tag{10}$$

Since $\hat{\beta} \le 0$, this clearly corresponds to cooling, rather than heating, of the electron beam. This is exactly as we would expect, since energetic particles tend to radiate more than less energetic ones. Less intuitively, we have a directional symmetry here, with equal contributions to cooling coming from the longitudinal and transverse directions (this is not strictly isotropic, though, since the two transverse directions contribute in total the same cooling as the single longitudinal direction). Moreover, there is complete isotropy within the transverse ϵ–λ plane, regardless of the polarisation of the laser.

How does the inclusion of quantum effects alter this picture? It is straightforward to show that switching from (2) to (4) modifies $\beta_\perp$ by the simple substitution $\tau \to \tilde{\tau}$. However, since $\chi = \hbar\sqrt{a_\epsilon^2 + a_\lambda^2}\,u_\phi/m$, $\beta_\parallel$ acquires an additional contribution from the derivative of g. Hence we have

$$\beta_\perp = g\hat{\beta}_\perp, \quad \beta_\parallel = \beta_\perp\left(1 + \frac{g'}{2g}\chi\right), \tag{11}$$

where the prime denotes differentiation with respect to χ. Since g decreases from 1 at $\chi = 0$ to 0 as $\chi \to \infty$, $g' < 0$, so the additional term tends to suppress the beam cooling.

Using the quantum dynamics, we still find that the beam cools, though there is a significant reduction in this effect. In addition, the symmetry between cooling in the longitudinal and transverse directions has been broken, with substantially more cooling in the latter, see Fig. 1. However, we once again find equal cooling in the two

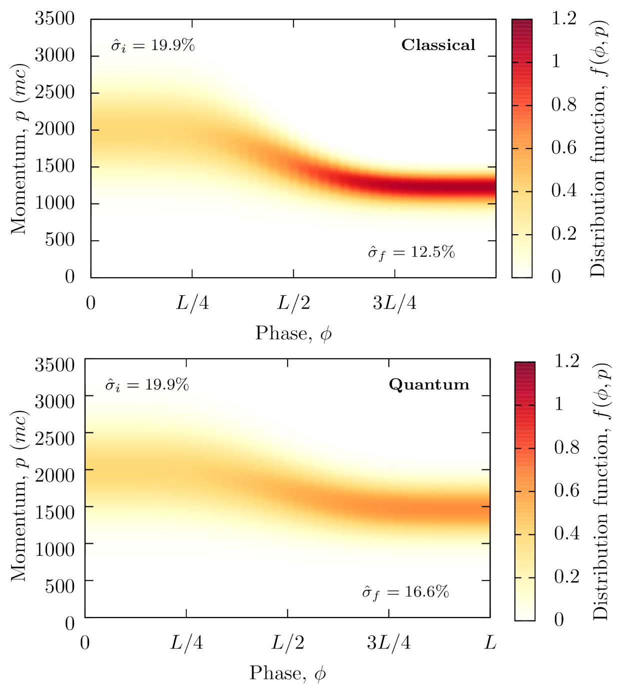

Figure 2: Evolution of the reduced distribution f for a beam of initial momentum spread 20% around an average of 1 GeV as it passes through a 27 fs laser pulse of intensity 2×10^{21} W/cm^2. The upper panel shows the classical result, where each electron satisfies (2), while the lower panel is the quantum-corrected version, satisfying (4).

transverse directions, irrespective of the polarisation of the laser in the ϵ–λ plane.

Given the substantial reduction in longitudinal relative to transverse beam cooling, it is pertinent to ask whether the former has an appreciable effect. To this end, we have investigated the solution to the longitudinal Vlasov equation,

$$\frac{df}{d\phi} + \beta_{\parallel} f = 0, \tag{12}$$

for the case of a 1 GeV electron beam with 20% momentum spread colliding with a 27 fs laser pulse of intensity 2×10^{21} W/cm^2. The results [7] are shown in Fig. 2. We clearly see that the quantum case exhibits substantially less beam cooling than the classical case (as well as less reduction in the average energy), but nevertheless the final momentum spread is significantly less than the initial value. While it would require a far greater computational overhead to compute the full distribution $\mathcal{F}(x, \dot{x})$, the change in its transverse spread can be readily determined from Figs. 1 and 2.

4 Conclusion

With the advent of a new breed of high power laser facility over the next few years, it is essential to understand how radiation reaction and quantum effects can influence the properties of relativistic electron beams interacting with ultra-intense laser pulses. We have explored a prime example of such effects, namely the radiative reduction of momentum spread, or beam cooling.

In the absence of radiation reaction, there is no beam cooling: the electron beam emerges from the laser pulse with the same momentum spread with which it started. The situation is changed when radiation reaction is included. According to the classical theory, the interaction with the laser pulse causes significant beam cooling, which is equally partitioned between the transverse and longitudinal directions. Including quantum effects changes things once again, with a substantial reduction in beam cooling, accompanied by a breaking of the symmetry between longitudinal and transverse directions, with the transverse beam cooling lying closer to the classical case, and the longitudinal cooling more resembling the case with no radiation reaction. Nevertheless, in all cases there is perfect isotropy within the transverse plane, the polarisation of the laser pulse playing no role at all.

Acknowledgements

We are grateful for support from the UK EPSRC (Grant EP/J018171/1); the ELI-NP Project; the European Commission FP7 projects Laserlab-Europe (Grant 284464) and EuCARD-2 (Grant 312453); and the Cockcroft Institute.

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