# Exploiting the self-similar nature of Raman and Brillouin scattering

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### Abstract

Raman and Brillouin amplification are two schemes for amplifying and compressing short laser pulses in plasma. Depending on the laser and plasma configurations, these schemes could potentially deliver the high-energy high-power pulses needed for inertial confinement fusion, especially fast-ignition fusion. Analytical self-similar models for both Raman and Brillouin amplification have already been derived [1, 2], but the consequences of this self-similar behavior are little known and hardly ever put to good use. In this paper, we will give an overview of the self-similar laws that govern the evolution of the probe pulse in Raman and Brillouin amplification. We will then show how these laws can be exploited, in particular regarding the parameters of the initial probe pulse, to control the properties of the final amplified probe and improve the efficiency of the process.

## 1 Introduction

In the context of Raman or Brillouin amplification, the term "self-similar" refers to the notion that the basic shape of the growing seed pulse does not change during the amplification process, while its amplitude and duration evolve according to well-defined scaling laws. For Raman amplification, the self-similar character of the seed pulse endows it with the following properties: (i) pulse amplitude is proportional to the interaction time $t$ [3–6], (ii) pulse duration is proportional to $1/t$, or bandwidth proportional to $t$ [4–7], (iii) pulse energy is proportional to $t$, or inversely proportional to its duration [8–13], (iv) the asymptotic self-similar "$\pi$-pulse" solution is an attractor solution, i.e. a "not quite ideal" seed pulse will reshape itself into an approximate $\pi$-pulse shape [1, 4, 7, 13–18], (v) in multi-dimensional simulations where the pulses have a finite transverse width, the seed pulse acquires a "bowed" shape [18–25]. Thus, evidence that the Raman-amplified seed pulse is self-similar is ubiquitous but not normally collated in a systematic way. Most of the above also applies to Brillouin amplification in the strong-coupling regime [2], although the scalings for the seed pulse duration and amplitude with interaction time are different.

## 2 Theory

We define $a_0$ and $a_1$ to be the (scaled) vector potential envelopes of pump and probe pulse respectively, $a_{0,1} \equiv (8.55 \times 10^{-10} g^{1/2} (I_{0,1}\lambda^2[\mathrm{Wcm^{-2}\mu m^2}])^{1/2}$, where $g = 1$ ($g = 1/2$) denotes linear (circular) polarisation, and $\delta n_e$ to be the envelope of the electron density fluctuations associated with the Langmuir wave. Let $\omega_0$ denote the pump laser frequency, and $n_e$ and $\omega_{pe}$ the background electron density and corresponding electron plasma frequency; $\omega_{pe} \ll \omega_0$ is assumed. The group velocity of the pump pulse is then given by $v_g = c^2 k_0/\omega_0 = c(1 - \omega_{pe}^2/\omega_0^2)^{1/2} \approx c$. The electron thermal velocity is defined via $v_e^2 = k_B T_e/m_e$ and $\lambda_D = v_e/\omega_{pe}$.

For Raman amplification, the envelope equations for pump, seed and plasma wave will take the following form:

$$(\partial/\partial t + v_g \partial/\partial x) a_0 = -\Gamma a_1 b, \tag{1}$$

$$(\partial/\partial t - v_g \partial/\partial x) a_1 = \Gamma a_0 b^*, \tag{2}$$

$$(\partial/\partial t + 3v_e^2 (k/\omega_{pe}) \partial/\partial x) b = \Gamma a_0 a_1^*, \tag{3}$$

where $\Gamma a_0$ denotes the Raman backscattering growth rate in homogeneous plasma and $b \equiv \alpha \delta n_e/n_e$ with $\alpha$ to be determined. Comparing these equations to the envelope equations by Forslund *et al.* [26] yields:

$$\Gamma\alpha = \omega_{pe}^2/(4\omega_0), \quad \Gamma/\alpha = c^2k^2/(4g\omega_{pe}), \tag{4}$$

where $k \approx 2k_0 \approx 2\omega_0/c$ is the wave number of the RBS Langmuir wave. It then follows that:

$$\Gamma = [\omega_0\omega_{pe}/(4g)]^{1/2}, \quad \alpha = \sqrt{g}(\omega_{pe}/\omega_0)^{3/2}/2. \tag{5}$$

Following the approach by Malkin, Shvets and Fisch [1], we define coordinates $\zeta = z/c + t$, $\tau = \Gamma^2 a_0^2 t$ and $\xi = 2\sqrt{\zeta\tau}$. Self-similar solutions to the above system can then be obtained in terms of $\xi$. In particular, the self-similar seed pulse will have a fixed duration $\xi_1 \approx 5$ (the quantity $\xi_M$ used elsewhere [1, 18] denotes the position of the seed pulse maximum rather than its duration).

By substituting $\zeta = \tau_1$ and $t = \tau_0/2$ into the definition of $\xi$, and using the energy conservation relation $a_0^2\tau_0 = a_1^2\tau_1$, we derive the following equations for the duration of the evolving seed pulse:

$$\Gamma^2 a_0^2 \tau_0 \tau_1 = \xi_1^2/2, \tag{6}$$

$$\Gamma a_1 \tau_1 = \xi_1/\sqrt{2}. \tag{7}$$

Scalings for the seed pulse duration and ampltiude follow immediately: $\tau_1(t) = \xi_1^2/(4a_0^2\Gamma^2 t)$ and $a_1(t) = 2\sqrt{2}a_0^2\Gamma t/\xi_1$.

The purpose of these equations is as follows. Eq. (6) allows one to derive scalings for $\tau_1(t)$ and $a_1(t)$, and also to tune the duration and amplitude of the final seed pulse via the intensity of the pump pulse [18]. Eq. (7) provides a relationship between seed pulse duration and amplitude that does not depend on the pump pulse at all. This is important for the preparation of the initial seed pulse in experiments: $\tau_1(0)$ and $a_1(0)$ are not independent parameters, but should ideally obey Eq. (7); if they do not do so, the seed pulse will first reshape itself until they do, and only amplify after that. Thus, tailoring the seed pulse according to (7) from the start will speed up the amplification process and increase its efficiency.

The energy density of a Langmuir wave is given by $\mathcal{E} = (1/2)n_e m_e(\omega_{pe}^2/k^2)(1+3k^2\lambda_D^2)||\delta n_e/n_e||^2$. The Manley-Rowe relation for the pump pulse and the RBS Langmuir wave then reads [26]: $a_0^2\omega_0 = g a_L^2\omega_{pe}$, or $a_0 = ||b||$, where $a_L = ||eE_L/(m_e\omega_{pe}c)|| = \omega_{pe}/(2\omega_0)||\delta n_e/n_e||$ is the scaled electric field of the Langmuir wave. The wave breaking limit for the RBS Langmuir wave is given by $a_L = \omega_{pe}/(2\omega_0)$, or $||\delta n_e/n_e|| = 1$ [1, 26, 28]. Wave breaking then happens for Raman amplification when $a_0 \geq \alpha$, with $\alpha$ as given above. Under the conditions $a_0 = \alpha$ and $\omega_{pe}\tau_1 \leq 1$, one finds that $a_1 \leq 2^{1/2}\xi_1(2g\alpha)^{1/3}$ and $\omega_0\tau_1 \geq g^{1/3}(2\alpha)^{-2/3}$ after amplification [4, 14, 29].

Brillouin scattering in the so-called weak-coupling regime [30–34] is very similar to Raman scattering and can be treated in the same way. We introduce $\omega_{pi} = \omega_{pe}\sqrt{Zm_e/m_i}$ and $c_s = v_e\sqrt{Zm_e/m_i}$. For $a_0^2 < 8g(\omega_0/\omega_{pe})^2 c_s v_e^2/c^3$, the electron pressure is the dominant restoring force and the plasma wave dispersion is not significantly affected by the beating between pump and seed pulses. In that case one can reuse equations (1)-(3) and only needs to replace $3v_e^2(k/\omega_{pe})$ by $c_s$ in (3). For backward Brillouin scattering, the ion-acoustic wave has wave number $k_s = 2k_0$ and frequency $\omega_s = c_s k_s = 2c_s k_0$. With these definitions, the equations for $\Gamma$ and $\alpha$ read:

$$\Gamma\alpha = \omega_{pe}^2/(4\omega_0), \quad \Gamma/\alpha = c^2c_s^2k_s^2/(4g\omega_s v_e^2), \tag{8}$$

and we find for $\Gamma$ and $\alpha$:

$$\Gamma = c\omega_{pe}\omega_s/(4v_e\sqrt{g\omega_0\omega_s}), \alpha = \sqrt{g}v_e\omega_{pe}/(c\sqrt{\omega_0\omega_s}). \tag{9}$$

With these definitions, all the above results for Raman amplification also apply to the weak-coupling Brillouin case, including Eqns. (6) and (7) and the seed pulse scalings. The energy density of an ion-acoustic wave is given by $\mathcal{E} = (1/2)v_e^2 m_e n_e(1+k^2\lambda_D^2)||\delta n/n_e||^2$ [26]. From this, we recover both the Manley-Rowe relation $a_0 = ||b||$ and the wave breaking threshold $a_0 \geq \alpha$.

For $a_0^2 > 8g(\omega_0/\omega_{pe})^2 c_s v_e^2/c^3$ or $\Gamma a_0/\omega_0 > c_s/c$, the ponderomotive pressure from the beating between pump and seed pulses will take over from the thermal pressure as the primary restoring force for the ion-acoustic wave. This regime is called *strong-coupling* Brillouin scattering. In this regime, the equation for the plasma wave needs to be replaced by:

$$\partial^2 b/\partial t^2 = -\alpha_{sc}c^2k^2\omega_{pi}^2/(2g\omega_{pe}^2)a_0a_1 = -\Gamma_{sc}^2 a_0 a_1. \tag{10}$$

From (1), (2) and (10) and using $k = 2k_0 = 2\omega_0 v_g/c^2$ as before, we find for $\Gamma_{sc}$ and $\alpha_{sc}$:

$$\Gamma_{sc}\alpha_{sc} = \omega_{pe}^2/(4\omega_0), \ \Gamma_{sc}^2/\alpha_{sc} = c^2k^2\omega_{pi}^2/(2g\omega_{pe}^2). \tag{11}$$

This leads to the following solutions [2, 26, 35]:

$$\Gamma_{sc} = [(v_g/c)^2\omega_{pi}^2\omega_0/(2g)]^{1/3} = (2\omega_s\Gamma^2)^{1/3}, \tag{12}$$

$$\alpha_{sc} = \omega_{pe}^2/(4\omega_0\Gamma_{sc}), \tag{13}$$

$$\Omega_{sc} = \omega_{sc} + i\gamma_{sc} = [(1+i\sqrt{3})/2]\Gamma_{sc}a_0^{2/3}. \tag{14}$$

Following the approach by Andreev *et al.* [2], we define $\tau = \Gamma_{sc}a_0^{2/3}t$, $\zeta = \Gamma_{sc}a_0^{2/3}(t+x/v_g)$ and $\xi = \zeta\sqrt{\tau}$. Again, self-similar solutions to the system (1), (2) and (10) can then be obtained in terms of $\xi$. In particular, it will be found that the self-similar seed pulse will have a fixed duration $\xi_1 \approx 2.5$.

By substituting $\zeta = \Gamma_{sc}\tau_1$ and $\tau = \Gamma_{sc}\tau_0/2$ into the definition of $\xi$ and using the energy conservation relation $a_0^2\tau_0 = a_1^2\tau_1$, we derive the following equations for the duration of the evolving seed pulse:

$$\Gamma_{sc}^3 a_0^2\tau_0\tau_1^2 = 2\xi_1^2, \tag{15}$$

$$\Gamma_{sc}^3 a_1^2\tau_1^3 = 2\xi_1^2. \tag{16}$$

Scalings for the seed pulse duration and ampltiude are then as follows [2]: $\tau_1(t) = \xi_1/(\Gamma_{sc}\sqrt{a_0^2\Gamma_{sc}t})$ and $a_1(t) = (2/\xi_1)^{1/2}(a_0^2\Gamma_{sc}t)^{3/4}$. The role of equations (15) and (16) is equivalent to the role of (6) and (7) for Raman amplification.

Plasma wave breaking in the strong-coupling regime of Brillouin scattering is not as straight-forward as for weak-coupling Brillouin or Raman scattering, because $\omega_{sc}$ is not constant throughout the process. In particular, there will be a sudden drop at the back of the probe pulse, where $\nabla^2(a_0 \cdot a_1)$ suddenly drops. Then the phase speed $\omega_{sc}/(2k_0)$ also drops while the ion oscillations maintain their amplitude, making the ion wave liable to breaking. This explains why Andreev *et al.* observed ion wave breaking just behind the growing probe pulse rather than inside it [2]. This will be studied in more detail elsewhere. In that light, the "wave breaking time" $t_{\mathrm{wb}} = (m_i/2m_e)^{1/2}/(a_0\omega_0)$ from Forslund *et al.* [26] likely refers to the time *after* the passage of the seed pulse rather than its duration.

Finally, we comment on the "bowed" shape of the amplified self-similar seed pulse. To correctly describe this shape, we need the time coordinate of the maximum of the seed pulse, $\tau_M$, rather than its duration, $\tau_1$.

From the definition of the self-similar coordinates, it follows that $\tau_M/\tau_1 = \xi_M^2/\xi_1^2 \approx 2$ for Raman amplification, while $\tau_M/\tau_1 = \xi_M/\xi_1$ for strong-coupling Brillouin amplification. Inserting these expressions into (6), (7), (15) or (16) and introducing $r$-dependent pulse amplitudes $a_{0,1}(r)$ provides a full description of the bowed shape via $\tau_M(r)$.

## 3 Discussion and conclusions

As shown before by Trines *et al.* [18], the self-similar equations involving both pump and probe parameters can be used to tune the final duration of the probe via the intensity of the pump. The other self-similar equations, which govern the evolution of the probe amplitude, are special in the sense that the pump pulse has been completely eliminated from them. They can be used in the preparation of the initial probe pulse as follows. As reported by Malkin *et al.* [1], the ideal "$\pi$-pulse" solution for the probe pulse in parametric amplification is a so-called *attractor*, which means that if the initial probe pulse does not have the ideal $\pi$-pulse shape, it will reshape itself first before it starts growing. If the probe pulse is prepared in such a way that its amplitude and duration obey the relevant self-similar relations from the start, this "reshaping" stage can be kept to a minimum, so the probe will start growing sooner and the amplification process will be more efficient.

While the scaling of probe pulse amplitude and duration with interaction time has been studied before [2, 7], the pump-independent relationships between probe amplitude and probe duration have not been derived explicitly to date. These relationships are important, however, when designing amplification experiments: if the probe pulse is shaped to obey these relationships at the start of the interaction, proper amplification will start sooner and the energy transfer will be more efficient.

Further scalings can be derived from the self-similar solutions in each case. This will be the subject of future study.

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